

Contents

Preface — VII

Acknowledgment — IX

Acronyms — XVII

Notation — XIX

Table of Units — XXIII

1 Introduction — 1

2 Fabrication of magneto-active polymer composites — 5

2.1 A general discussion on MAP fabrication — 5

2.2 Characterisable MAP: From lab to fab — 10

3 Experimental apparatus and testing procedure — 15

3.1 Parallel-plate rotational rheometer — 16

3.1.1 Stress-controlled deformation — 18

3.1.2 Magneto-rheological device — 21

3.1.3 Influence of rotor geometry on magnetic field — 23

3.1.4 Caveats to consider when performing experiments — 26

3.2 Experimental methodology — 31

4 Magneto-mechanical characterisation of magneto-active polymer composites — 35

4.1 Unfilled matrix (PDMS) — 35

4.2 Microstructure developed in the cured filled MAP — 37

4.3 Strain amplitude dependence of the composite MAP — 39

4.3.1 Response to mechanical loading — 39

4.3.2 Response to magneto-mechanical loading — 42

4.3.3 Influence of rotor geometries on the response of isotropic MAPs — 45

4.3.4 Modelling of isotropic MAPs at a fixed frequency — 46

4.4 Frequency dependence of the composite MAP — 47

4.4.1 Response to mechanical loading — 49

4.4.2 Response to magneto-mechanical loading — 52

5 Introduction to continuum magneto-mechanics — 54

5.1 Continuum setting — 54

5.1.1	Continuum domain —	54
5.1.2	Kinematics —	55
5.1.3	Line, area and volume transformations —	56
5.1.4	Time derivatives —	57
5.1.5	Fundamentals of electromagnetism —	58
5.2	Continuum theorems for materials with discontinuities —	63
5.2.1	Control volumes with surface discontinuities —	64
5.2.2	Control surfaces with line discontinuities —	68
5.3	Governing equations —	70
5.3.1	Balance laws —	72
5.3.2	Spatial relationships —	76
5.3.3	Summary of balance laws —	77
5.3.4	Ponderomotive force and moment —	82
5.3.5	Formulations for magnetic potentials —	84
5.3.6	Weak formulation of conservation laws —	86
5.3.7	Variational formulations —	88
5.4	Thermodynamics —	93
5.4.1	First law of thermodynamics: Energy balance —	94
5.4.2	Second law of thermodynamics: Entropy balance —	96
5.4.3	Parameterisation of isothermal energy functions —	98
6	General aspects of computational simulation of coupled problems —	100
6.1	Finite element discretisation —	100
6.1.1	Displacement field —	101
6.1.2	Magnetic vector potential field —	105
6.1.3	Magnetic scalar potential field —	107
6.1.4	Summary of finite element implementation —	109
6.1.5	Tools automating the computation of finite element linearisations and constitutive model tangent moduli —	110
6.2	Evaluation of definite integrals —	111
6.3	Solution of a time/load increment —	113
6.3.1	Solution to the time-independent non-linear problem using the Newton–Raphson method —	114
6.3.2	Formation and solution of the discrete system of linear equations —	114
6.3.3	Special considerations when using the magnetic scalar potential formulation —	116
7	Constitutive modelling —	118
7.1	Preliminaries to magneto-mechanical energy functions —	120
7.1.1	Volumetric-isochoric split —	120
7.1.2	Invariants for isotropic media —	121

7.1.3	Transverse isotropy — 122
7.2	Viscomagneto-viscoelasticity — 123
7.3	Transverse isotropy and particle chain dispersion — 133
8	Phenomenological modelling of the curing process — 142
8.1	A continuum framework for the curing of polymers — 145
8.1.1	Curing in viscomagneto-viscoelastic materials — 147
8.1.2	Curing with field-sensitive shrinkage effects — 149
9	Homogenisation — 157
9.1	First-order homogenisation of magneto-coupled materials — 159
9.1.1	Hill's condition on the equality of micro- and macro-scale virtual powers — 162
9.1.2	Consistent boundary conditions arising from the equality of virtual power — 164
9.1.3	Computation of consistent tangent moduli for the macro-scale problem — 174
9.1.4	Homogenisation of curing using the Mori–Tanaka method — 178
9.2	The stochastic finite element method — 184
9.2.1	Extension into stochastic dimensions — 185
9.2.2	Defining material discontinuities using level set functions — 188
9.2.3	Basis function selection — 189
10	Modelling and computational simulation at the micro-scale — 200
10.1	Single particle representative volume element problem — 200
10.1.1	Solution accuracy and finite element discretisation — 201
10.1.2	Computation of magnetic forces and torques — 206
10.2	Micro-structural studies of MAP compositions — 211
10.2.1	Full resolution simulation of a prototype magnetostatic microstructural model — 212
10.2.2	Influence of microstructural organisation in a representative volume element on the response characteristics of a prototype magnetoelastic material — 215
11	Modelling and computational simulation at the macro-scale — 224
11.1	Macro- to micro-scale transition using the FE^2 approach — 224
11.2	Immersion of magnetic bodies in free space — 228
11.2.1	Mesh motion in the free space — 229
11.3	Mixed variational approach for quasi-incompressible media — 238
12	Further reading — 251

A Identities — 255

- A.1 Operation identities — 255
- A.2 Generic differential identities — 257
- A.3 Differential and rate identities: Continuum mechanics — 262

B Calculus — 265

- B.1 Microscopic theorems — 265
- B.2 Continuum theorems — 266
- B.2.1 Materials without discontinuities — 268
- B.2.2 Materials with discontinuities — 271
- B.3 Continuum identities — 275

C Derivations and proofs — 277

- C.1 Fundamentals of electromagnetism — 277
- C.1.1 Electrostatics — 277
- C.1.2 Magnetostatics — 278
- C.2 Continuum mechanics for magnetoelasticity — 279
- C.3 Stress tensors — 283
- C.3.1 Definitions — 283
- C.3.2 Divergences — 285
- C.3.3 Jumps — 286
- C.4 Ponderomotive forces and tractions — 287
- C.4.1 Definitions of the Lorentz forces — 287
- C.5 Thermomechanical and electromagnetic balance laws — 290
- C.5.1 Derivation of spatial statement of Maxwell's equations in a non-relativistic Eulerian reference frame — 290
- C.5.2 Derivation of spatial statement of mechanical conservation laws (for magnetostatic systems) — 294
- C.5.3 Derivation of additional balance and jump identities for potentials — 298
- C.5.4 Transformation of conservation laws to their referential description — 299
- C.5.5 Weak formulation of quasi-static balance of linear momentum — 306
- C.6 Legendre transformations — 307
- C.7 Thermodynamics — 313
- C.7.1 Work performed by the Lorentz forces — 313
- C.7.2 Boundary contributions to external power — 315
- C.7.3 Combined contribution of external mechanical and electromagnetic powers — 316
- C.8 Homogenisation — 317
- C.8.1 Relationship between macroscopic and microscopic field quantities — 317

C.8.2	The Hill–Mandel condition —	319
C.8.3	Algorithmically consistent tangent moduli —	321
C.9	Constitutive modelling —	325
C.9.1	Free energy function derivatives —	325
C.9.2	Invariants —	325
C.9.3	Invariant derivatives —	326
C.9.4	Free space stored energy function derivatives —	327
C.9.5	Volumetric / deviatoric split of free energy function —	328
C.10	Time integrators for rate-dependent materials described by internal variables —	329

Bibliography — 333

Image reproduction — 367

Index — 369