

Contents

Part I Foundations

Chapter 1: Semirings and Formal Power Series

<i>Manfred Droste and Werner Kuich</i>	3
1 Introduction	3
2 Monoids and Semirings	5
3 Formal Power Series	12
4 Matrices	17
5 Cycle-Free Linear Equations	22
References	26

Chapter 2: Fixed Point Theory

<i>Zoltán Ésik</i>	29
1 Introduction	29
2 Least Fixed Points	30
3 Conway Theories	38
4 Iteration Theories	41
5 Unique Fixed Points	44
6 Fixed Points of Linear Functions	46
6.1 Inductive $\ast$ -Semirings	51
6.2 Complete Semirings	52
6.3 Iterative Semirings	53
7 Fixed Points of Affine Functions	54
7.1 Complete Semiring–Semimodule Pairs	59
7.2 Bi-inductive Semiring–Semimodule Pairs	61
References	62

Part II Concepts of Weighted Recognizability

Chapter 3: Finite Automata

<i>Zoltán Ésik and Werner Kuich</i>	69
1 Introduction	69
2 Finite Automata over Semirings	70
2.1 Finite Automata over Arbitrary Power Series Semirings	72
2.2 Finite Automata over Conway Semirings	75
2.3 Finite Linear Systems	81
3 Finite Automata over Quemirings	83
3.1 Semiring–Semimodule Pairs and Quemirings	85
3.2 Finite Automata over Quemirings and a Kleene Theorem	91
3.3 Finite Linear Systems over Quemirings	99
References	103

Chapter 4: Rational and Recognisable Power Series

<i>Jacques Sakarovitch</i>	105
1 Introduction	106
2 Rational Series and Weighted Rational Expressions	107
2.1 Series over a Graded Monoid	107
2.2 Rational Series	114
3 Weighted Automata	122
3.1 The Behaviour of a Weighted Automaton	122
3.2 The Fundamental Theorem of Automata	126
3.3 Conjugacy and Covering of Automata	132
4 Recognisable Series and Representations	138
4.1 The Family of Recognisable Series	138
4.2 Other Products on Recognisable Series	141
4.3 Series on a Product of Monoids	145
5 Series over a Free Monoid	151
5.1 The Representability Theorem	152
5.2 Reduced Representations	157
5.3 Applications of the Reduction of Representations	161
6 Support of Rational Series	165
7 Notes	168
7.1 General Sources	168
7.2 Notes to Sect. 2: Rational Series	169
7.3 Notes to Sect. 3: Weighted Automata	169
7.4 Notes to Sect. 4: Recognisable Series	170
7.5 Notes to Sect. 5: Series over a Free Monoid	170
7.6 Notes to Sect. 6: Support of Rational Series	171
References	171

Chapter 5: Weighted Automata and Weighted Logics

<i>Manfred Droste and Paul Gastin</i>	175
1 Introduction	175
2 MSO-Logic and Weighted Automata	178
3 Weighted Logics	181
4 Unambiguous Formulas	185
5 Definability Equals Recognizability	189
6 Locally Finite Semirings	194
7 Weighted Automata on Infinite Words	197
8 Weighted Logics on Infinite Words	203
9 Conclusions and Open Problems	206
References	208

Chapter 6: Weighted Automata Algorithms

<i>Mehryar Mohri</i>	213
1 Introduction	214
2 Preliminaries	214
2.1 Semirings	214
2.2 Weighted Transducers and Automata	215
3 Shortest-Distance Algorithms	217
3.1 All-Pairs Shortest-Distance Problems	218
3.2 Single-Source Shortest-Distance Problems	222
4 Rational Operations	223
5 Elementary Unary Operations	226
6 Fundamental Binary Operations	226
6.1 Composition	226
6.2 Intersection	231
6.3 Difference	231
7 Optimization Algorithms	233
7.1 Epsilon-Removal	233
7.2 Determinization	237
7.3 Weight Pushing	241
7.4 Minimization	243
7.5 Synchronization	246
8 Conclusion	250
References	250

Part III Weighted Discrete Structures**Chapter 7: Algebraic Systems and Pushdown Automata**

<i>Ion Petre and Arto Salomaa</i>	257
1 Introduction	257
2 Auxiliary Notions and Results	258
3 Algebraic Power Series	261

3.1	Definition and Basic Reductions	261
3.2	Interconnections with Context-Free Grammars	265
3.3	Normal Forms	267
3.4	Theorems of Shamir and Chomsky–Schützenberger	271
4	Transductions	275
5	Pushdown Automata	279
5.1	Pushdown Transition Matrices	279
5.2	$S\langle\langle\Sigma^*\rangle\rangle$ -Pushdown Automata	281
5.3	Equivalence with Algebraic Systems	284
6	Other Topics	287
	References	288

Chapter 8: Lindenmayer Systems

	<i>Juha Honkala</i>	291
1	Introduction	291
2	Iterated Morphisms and Rational Series	292
3	Lindenmayerian Algebraic Series	296
4	D0L Power Series	302
5	Other Power Series Generalizations of L Systems	307
	References	309

Chapter 9: Weighted Tree Automata and Tree Transducers

	<i>Zoltán Fülöp and Heiko Vogler</i>	313
1	Introduction	314
2	Preliminaries	316
2.1	General Notation	316
2.2	Trees	316
2.3	Algebraic Concepts	317
2.4	Tree Series	320
3	Weighted Tree Automata	320
3.1	Bottom-up Tree Automata	320
3.2	Recognizable Tree Series	321
3.3	Closure Properties	326
3.4	Support of Recognizable Tree Series	328
3.5	Determinization of Weighted Tree Automata	330
3.6	Pumping Lemmata and Decidability	331
3.7	Finite Algebraic Characterizations of Recognizable Tree Series	335
3.8	Equational Tree Series	342
3.9	Rational Tree Series	347
3.10	MSO-Definable Tree Series	349
3.11	Other Models Related to Recognizable Tree Series	353
3.12	Further Results	360
4	IO-Substitution and Tree Series Transformations	361
5	Weighted Tree Transducers	364
5.1	Tree Transducers	364

5.2	The Basic Model	365
5.3	Restricted Models	370
5.4	Composition and Decomposition	374
5.5	The Inclusion Diagram of Some Fundamental wtt Classes	386
5.6	Hierarchies	388
5.7	Further Models of Weighted Tree Transducers	391
5.8	Further Results	394
	References	394

Chapter 10: Traces, Series-Parallel Posets, and Pictures:

A Weighted Study

	<i>Ina Fichtner, Dietrich Kuske, and Ingmar Meinecke</i>	405
1	Introduction	406
2	Traces	407
2.1	Weighted Distributed Systems	407
2.2	Other Formalisms: Presentations, Expressions, and Logics	411
2.3	Relating the Formalisms	413
2.4	History and Overview	421
3	Series-Parallel Posets	423
3.1	Series-Parallel Posets and Bisemirings	423
3.2	Weighted Branching Automata	425
3.3	Rationality	427
3.4	The Hadamard Product	433
3.5	History and Overview	435
4	Pictures	436
4.1	Pictures and Weighted Picture Automata	436
4.2	Other Formalisms: Expressions and Logics	438
4.3	Relating the Formalisms	440
4.4	Decidability Issues	444
4.5	History and Overview	445
	References	446

Part IV Applications

Chapter 11: Digital Image Compression

	<i>Jürgen Albert and Jarkko Kari</i>	453
1	Introduction	453
2	Image Types	454
3	Weighted Finite Automata and Multi-resolution Images	457
4	Drawing WFA Images	459
5	An Encoding Algorithm	460
6	Practical Image Compression Using WFA	463
7	Weighted Finite Transducers (WFT)	469
8	Parametric Weighted Finite Automata (PWFA)	472

9	Conclusions and Open Problems	476
	References	477

Chapter 12: Fuzzy Languages

	<i>George Rahonis</i>	481
1	Introduction	481
2	Lattices and Fuzzy Languages	483
3	Fuzzy Recognizability over Bounded Distributive Lattices	486
	3.1 Fuzzy Recognizability over Finite Words	487
	3.2 Fuzzy Recognizability over Infinite Words	495
	3.3 Multi-valued MSO Logic	500
4	Fuzzy Languages: An Overview	505
	4.1 Fuzzy Languages over ℓ -Monoids	506
	4.2 Fuzzy Languages over Residuated Lattices	507
	4.3 Fuzzy Automata with Outputs	509
	4.4 Fuzzy Abstract Families of Languages	509
	4.5 Fuzzy Tree Languages	509
5	Applications	510
	References	513

Chapter 13: Model Checking Linear-Time Properties of Probabilistic Systems

	<i>Christel Baier, Marcus Größer, and Frank Ciesinski</i>	519
1	Introduction	519
2	Markov Decision Processes	526
3	Maximal Reachability Probabilities	533
4	Model Checking ω -Regular Properties	538
5	Partial Order Reduction	547
6	Partially Observable MDPs	557
7	Conclusion	559
8	Appendix	560
	References	563

Chapter 14: Applications of Weighted Automata in Natural Language Processing

	<i>Kevin Knight and Jonathan May</i>	571
1	Background	571
2	WFST Techniques for Natural Language Processing	572
	2.1 Example 1: Transliteration	573
	2.2 Example 2: Translation	578
	2.3 Language Modeling	582
3	Applications of Weighted String Automata	583
	3.1 Language Translation	584
	3.2 Speech Recognition	584
	3.3 Lexical Processing	585
	3.4 Tagging	585

3.5	Summarization	586
3.6	Optical Character Recognition	586
4	Applications of Weighted Tree Automata	586
4.1	Open Problems	590
5	Conclusion	591
	References	591
Index		597