Contents

Introduction	1
1 Hypergroups and their measure algebras	6
1.1 Definition and general constructions	6
1.2 Translation and convolution	15
1.3 Invariant measures	28
1.4 Convolution of functions	41
1.5 Subhypergroups and double coset hypergroups	49
1.6 Idempotent measures and multipliers Notes	61 69
110103	07
2 The dual of a commutative hypergroup	74
2.1 Representations and Fourier transforms	74
2.2 The dual space in the commutative case	78
2.3 Modification of the convolution	101
2.4 The dual hypergroup	110
2.5 Support of the Plancherel measure	120
Notes	128
3 Some special classes of hypergroups	133
3.1 Polynomial hypergroups in several variables	133
3.2 Polynomial hypergroups in one variable	147
3.3 Examples of polynomial hypergroups in one variable	163
3.4 One-dimensional hypergroups	189
3.5 Sturm-Liouville hypergroups	201
3.6 Characterization of Pontryagin hypergroups	248
Notes	255
4 Positive and negative definite functions and measures	258
4.1 Positive definite functions	258
4.2 The Lévy continuity theorem	276
4.3 Positive definite measures	283
4.4 Negative definite functions	292
4.5 The Lévy-Khintchine representation	304
Notes	313



vi Contents

5 Convolution semigroups and divisibility of measures	317
5.1 Convergence of nets of measures	317
5.2 Convolution semigroups of measures	329
5.3 Embedding infinitely divisible measures	348
5.4 Factorization on hermitian hypergroups	355
Notes	379
6 Transience of convolution semigroups	382
6.1 The dichotomy theorem for random walks	382
6.2 The generalized Chung-Fuchs criterion	393
6.3 Transience and renewal of convolution semigroups	400
6.4 Characterization of potential measures	409
6.5 Invariant Dirichlet forms	421
Notes	443
7 Randomized sums of hypergroup-valued random variables	446
7.1 Concretization of hypergroups	446
7.2 Moment functions	452
7.3 Strong laws of large numbers	468
7.4 Central limit theorems	487
7.5 Invariance principles	515
Notes	530
8 Further topics	534
8.1 Towards a structure theory for hypergroups	534
8.2 Towards a theory of stationary random fields over hypergroups	546
Bibliography	553
Examples	581
Symbols	589
Index	597