

Contents

1	Introduction: Leibniz, Turing, Zuse and Beyond	1
	Leibniz' Vision of Divine Automata	1
	The Church–Turing Thesis	2
	The Birth of Cellular Automata	5
	The Zuse–Fredkin Thesis	6
	A New Kind of Science?	8
	Complex Dynamical Systems	9
	Beyond the Church–Turing Thesis?	13
	References	15
2	Simplicity in the Universe of Cellular Automata	17
	From Simple Local Rules to Global Complex Patterns	17
	Cellular Automata as Dynamical Systems	19
	Digital Dynamics with Difference Equations	22
	References	24
3	Complexity in the Universe of Cellular Automata	25
	Complexity Index of Cellular Automata	25
	Analytical Geometry of Boolean Cubes	27
	From Simple Building Blocks to Complex Compositions	27
	Computational Complexity and Universal Computability	29
	References	30
4	Symmetry in the Universe of Cellular Automata	31
	Local Equivalence of Cellular Automata	31
	Global Equivalence of Cellular Automata	33
	Symmetry with Global Transformations	35
	Symmetry of Left–Right Transformation T^\dagger	35
	Symmetry of Global Complementation \bar{T}	37
	Symmetry of Left–Right Complementation T^*	38

Global Symmetry of Klein's Vierergruppe \mathcal{V}	39
Local Symmetry Classes of Cellular Automata	40
The Holy Grail of Symmetry and Computability	42
References	43
5 Attractors in the Universe of Cellular Automata	45
Transient Regime and Basin of Attractors	45
Characteristic Functions of Cellular Automata	47
Poincaré Return Maps of Cellular Automata	50
Lameray Diagrams of Cellular Automata	51
Power Spectrum of Cellular Automata	52
Invertible Attractors	55
Bernoulli Shifts of Cellular Automata	56
Bernoulli Shifts and Coin-Toss Experiments	58
Fractality of Cellular Automata	59
Gardens of Eden and Isles of Eden	61
Basin Trees of Attractors	62
References	65
6 Time in the Universe of Cellular Automata	67
Time Reversal Test of Cellular Automata	67
Time Reversibility and Arrow of Time	69
Time Reversibility and Invertibility	70
Random Walks, Time and Cellular Automata	73
References	73
7 Matter in the Universe of Cellular Automata	75
Symmetries in the Universe of Physics	75
Symmetries in the Universe of Automata	77
Expansion in the Universe of Physics and Automata	81
Quantum Matter and Quantum Information	82
The Universe of Quantum Cellular Automata	83
References	85
8 Life and Brain in the Universe of Cellular Automata	87
Self-Organization and Emergence in Cellular Automata	87
Systems Biology and Cellular Automata	89
Brain Research and Cellular Automata	93
Brains of Memristors?	100
References	102
9 Outlook: Is the Universe a Computer?	105
References	108