

Contents

List of Figures	xv
List of Tables	xix
List of Algorithms	xix
Glossary	xxiii
Abbreviations	xxix
1 Introduction	1
1.1 Uncertainty quantification	2
1.1.1 Aleatory uncertainty	2
1.1.2 Epistemic uncertainty	4
1.1.3 Mixed uncertainty	5
1.2 Uncertainty propagation	5
1.2.1 Probabilistic approaches	6
1.2.2 Possibilistic approaches	7
1.2.3 Hybrid approaches	7
1.3 Outline	9
2 Describing Uncertain Parameters	11
2.1 Probability theory for aleatory uncertain parameters	12
2.1.1 Random variables	13
2.1.2 Random fields	17
2.2 Interval theory for epistemic uncertain parameters	20
2.2.1 Interval variables	20
2.2.2 Interval fields	23
2.3 Probability box theory for mixed uncertainties	24
2.3.1 Imprecise random variables	24
2.3.2 Imprecise random fields	26
3 Propagating Imprecise Random Fields	29
3.1 Karhunen-Loève expansion to discretise random fields	30
3.1.1 Truncation error of the Karhunen-Loève expansion	32
3.1.2 Correlation structures	33

3.1.3	Solution of the Fredhold integral equation	38
3.2	Interval analysis to consider interval variables	42
3.2.1	Vertex propagation	43
3.2.2	Global optimisation	43
3.3	Probability bounds analysis for imprecise random fields	44
3.3.1	Computational cost	49
3.3.2	Influence of the truncation error	50
3.3.3	Influence of the spatial discretisation	52
4	Solid Mechanics	53
4.1	Continuum mechanical framework	54
4.1.1	Balance equations	55
4.1.2	Constitutive theory	57
4.2	Material descriptions in terms of small strains	59
4.2.1	Linear-elasticity	59
4.2.2	Elasto-plasticity	60
4.3	Non-linear finite element method	62
5	Sampling Based Stochastic Finite Element Methods	67
5.1	Sampling approaches	68
5.1.1	Monte Carlo simulation	69
5.1.2	Stochastic collocation method	70
5.1.3	Polynomial chaos expansion	74
5.2	Comparison in terms of solid mechanical applications	78
5.2.1	One-dimensional example including high stochastic dimensions	79
5.2.2	Two-dimensional example including a non-linear model	82
5.3	Summary and concluding remarks	85
6	Limit Representation of Imprecise Random Fields	87
6.1	Investigation on the correlation length	89
6.2	Decoupled interpolation approach	93
6.2.1	General idea	94
6.2.2	Algorithmic treatment	96
6.2.3	Error measures	101
6.3	Study on one-dimensional random fields	101
6.3.1	Bending beam with linear-elastic material behaviour	101
6.3.2	Tensile bar with elasto-plastic material behaviour	110
6.4	Application to a two-dimensional random field	115
6.4.1	Investigation on the convergence behaviour	116
6.4.2	Results obtained by the interpolation approach	119
6.5	Summary and concluding remarks	120
7	Conclusion and Perspectives	123

Bibliography	127
Curriculum Vitae	I
Research and Seminar Reports	V