

René Dáger and Enrique Zuazua

# Wave Propagation, Observation and Control in $1-d$ Flexible Multi-Structures

 Springer

---

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Preliminaries</b>	<b>9</b>
2.1	The Elastic String	9
2.2	Networks of Strings	14
2.2.1	Elements on Graphs	14
2.2.2	Equations of Motion for Networks	15
2.3	The Control Problem	21
2.3.1	Basic Definitions	21
2.3.2	An Equivalent Formulation of the Control Problem	22
2.4	A Controllability Theorem and its Limitations	24
<b>3</b>	<b>Some Useful Tools</b>	<b>25</b>
3.1	D'Alembert Formula and Boundary Observability of the $1 - d$ Wave Equation	25
3.1.1	D'Alembert Formula	25
3.1.2	Boundary Observability of the $1 - d$ Wave Equation	27
3.2	HUM	28
3.2.1	Description of the Method	28
3.2.2	Application to the Control of Networks	33
3.3	The Method of Moments	36
3.3.1	Description of the Method	36
3.3.2	Application to the Control of Networks	40
3.4	Riesz Bases and Ingham-Type Inequalities	45
3.4.1	Riesz Bases	45
3.4.2	Generalized Ingham Theorems	45
3.4.3	A New Inequality	49
<b>4</b>	<b>The Three String Network</b>	<b>53</b>
4.1	The Three String Network with Two Controlled Nodes	53
4.1.1	Equations of Motion of the Network	53

4.1.2	The Control Problem .....	55
4.2	A Simpler Problem: Simultaneous Control of Two Strings ....	57
4.2.1	Identification of Controllable Subspaces .....	61
4.3	The Three String Network with One Controlled Node .....	67
4.4	An Observability Inequality .....	69
4.5	Properties of the Sequence of Eigenvalues .....	76
4.6	Observability of the Fourier Coefficients .....	80
4.7	Study of the Weights $c_n$ .....	81
4.8	Relation with the Simultaneous Control of Two Strings .....	87
4.9	Lack of Observability in Small Time .....	90
4.10	Application of the Method of Moments to Control .....	96
<b>5</b>	<b>General Trees .....</b>	<b>103</b>
5.1	Notations and Statement of the Problem .....	104
5.1.1	Notations for Graphs .....	104
5.1.2	Equations of Motion .....	105
5.2	The Operators $\mathcal{P}$ and $\mathcal{Q}$ .....	108
5.2.1	A Tree Formed by a Single String .....	108
5.2.2	Operators of Type $S$ .....	108
5.2.3	Construction of $\mathcal{P}$ and $\mathcal{Q}$ in the General Case .....	114
5.2.4	The Action of $\mathcal{P}$ and $\mathcal{Q}$ at the Interior Nodes .....	116
5.2.5	Action of $\mathcal{P}$ and $\mathcal{Q}$ on the Solution .....	118
5.3	The Main Observability Result .....	119
5.4	Relation Between $\mathcal{P}$ and $\mathcal{Q}$ and the Spectrum .....	123
5.4.1	The Eigenvalue Problem .....	124
5.4.2	Further Properties of $\mathcal{P}$ and $\mathcal{Q}$ .....	129
5.5	Observability Results .....	131
5.5.1	Weighted Observability Inequalities .....	131
5.5.2	Non-degenerate Trees .....	133
5.5.3	On the Set of Non-degenerate Trees .....	135
5.6	Consequences Concerning Controllability .....	137
5.7	Simultaneous Observability and Controllability of Networks ..	138
5.8	Examples .....	142
5.8.1	The Star-Shaped Network with $n$ Strings .....	142
5.8.2	Simultaneous Control of $n$ Strings .....	144
5.8.3	A Non Star-Shaped Tree .....	146
<b>6</b>	<b>Some Observability and Controllability Results for General Networks .....</b>	<b>149</b>
6.1	Spectral Control of General Networks .....	150
6.1.1	Asymptotic Behavior of the Eigenfunctions .....	150
6.1.2	Application to Control .....	154
6.2	Colored Networks .....	157
6.3	Optimality of Theorem 3.2.7 .....	161
6.3.1	Simultaneous Control of Serially Connected Strings ....	162

<b>7</b>	<b>Simultaneous Observation and Control from an Interior Region</b>	167
7.1	Simultaneous Interior Control of Two Strings	168
7.1.1	Statement of the Problem	168
7.1.2	Control of Strings with Different Densities	170
7.1.3	Control of Strings with Equal Densities	174
7.2	Simultaneous Control on the Whole Domain	177
<b>8</b>	<b>Other Equations on Networks</b>	183
8.1	The Heat Equation	184
8.2	The Schrödinger Equation	187
8.3	A Model of Network for Beams	192
<b>9</b>	<b>Final Remarks and Open Problems</b>	197
9.1	Brief Description of the Main Results of the Book	197
9.1.1	Networks of Strings	197
9.1.2	Simultaneous Control of Strings	198
9.1.3	Other Equations on Networks	199
9.2	Future Lines of Research and Open Problems	199
	<b>Some Consequences of Diophantine Approximation Theorems</b>	205
	<b>References</b>	213
	<b>Index</b>	221