

Contents

Preface — VII

1 Basic principles of tomography — 1

- 1.1 Tomography — 1
- 1.2 Projection — 3
- 1.3 Image reconstruction — 6
- 1.4 Backprojection — 8
- 1.5 Mathematical expressions — 9
 - 1.5.1 Projection — 9
 - 1.5.2 Backprojection — 10
 - 1.5.3 The Dirac δ -function — 12
- 1.6 Worked examples — 14
- 1.7 Summary — 18
 - Problems — 18
 - Bibliography — 19

2 Parallel-beam image reconstruction — 20

- 2.1 Fourier transform — 20
- 2.2 Central slice theorem — 21
- 2.3 Reconstruction algorithms — 23
 - 2.3.1 Method 1 — 24
 - 2.3.2 Method 2 — 24
 - 2.3.3 Method 3 — 26
 - 2.3.4 Method 4 — 26
 - 2.3.5 Method 5 — 26
 - 2.3.6 Method 6 — 27
- 2.4 A computer simulation — 28
- 2.5 ROI reconstruction with truncated projections — 29
- 2.6 Mathematical expressions — 34
 - 2.6.1 The Fourier transform and convolution — 34
 - 2.6.2 The Hilbert transform and the finite Hilbert transform — 35
 - 2.6.3 Proof of the central slice theorem — 37
 - 2.6.4 Derivation of the FBP algorithm — 38
 - 2.6.5 Expression of the convolution backprojection algorithm — 39
 - 2.6.6 Expression of the Radon inversion formula — 40
 - 2.6.7 Derivation of the backprojection-then-filtering algorithm — 40
 - 2.6.8 Expression of the derivative-backprojection-Hilbert transform algorithm — 41
 - 2.6.9 Derivation of the backprojection-derivative-Hilbert transform algorithm — 42

2.7	Worked examples —	42
2.8	Summary —	49
	Problems —	50
	Bibliography —	50
3	Fan-beam image reconstruction —	52
3.1	Fan-beam geometry and the point spread function —	52
3.2	Parallel-beam to fan-beam algorithm conversion —	55
3.3	Short scan —	57
3.4	Mathematical expressions —	59
3.4.1	Derivation of a filtered backprojection fan-beam algorithm —	60
3.4.2	A fan-beam algorithm using the derivative and the Hilbert transform —	61
3.4.3	Expression for the Parker weights —	63
3.4.4	Errors caused by finite bandwidth implementation —	64
3.5	Worked examples —	65
3.6	Summary —	68
	Problems —	69
	Bibliography —	69
4	Transmission and emission tomography —	71
4.1	X-ray computed tomography —	71
4.2	Positron emission tomography and single-photon emission computed tomography —	75
4.3	Noise propagation in reconstruction —	78
4.3.1	Noise variance of emission data —	78
4.3.2	Noise variance of transmission data —	79
4.3.3	Noise propagation in an FBP algorithm —	79
4.4	Attenuation correction for emission tomography —	80
4.4.1	PET —	80
4.4.2	SPECT: Tretiak–Metz FBP algorithm for uniform attenuation —	81
4.4.3	SPECT: Inouye’s algorithm for uniform attenuation —	83
4.5	Mathematical expressions —	85
4.5.1	Expression for Tretiak–Metz FBP algorithm —	85
4.5.2	Derivation for Inouye’s algorithm —	85
4.5.3	Rullgård’s derivative-then-backprojection algorithm for uniform attenuation —	87
4.5.4	Novikov–Natterer FBP algorithm for nonuniform attenuation SPECT —	88
4.6	Worked examples —	89

4.7	Summary — 91
	Problems — 91
	Bibliography — 92
5	Three-dimensional image reconstruction — 94
5.1	Parallel line-integral data — 94
5.1.1	Backprojection-then-filtering — 96
5.1.2	Filtered backprojection — 98
5.2	Parallel plane-integral data — 98
5.3	Cone-beam data — 100
5.3.1	Feldkamp's algorithm — 101
5.3.2	Grangeat's algorithm — 102
5.3.3	Katsevich's algorithm — 104
5.4	Mathematical expressions — 108
5.4.1	Backprojection-then-filtering for parallel line-integral data — 108
5.4.2	FBP algorithm for parallel line-integral data — 109
5.4.3	Three-dimensional Radon inversion formula (FBP algorithm) — 112
5.4.4	Three-dimensional backprojection-then-filtering algorithm for Radon data — 113
5.4.5	Feldkamp's algorithm — 113
5.4.6	Tuy's relationship — 114
5.4.7	Grangeat's relationship — 117
5.4.8	Katsevich's algorithm — 119
5.5	Radon transform and ray transform in n dimensions — 125
5.6	Worked examples — 126
5.7	Summary — 129
	Problems — 130
	Bibliography — 130
6	Iterative reconstruction — 133
6.1	Solving a system of linear equations — 133
6.2	Algebraic reconstruction technique — 138
6.3	Gradient descent algorithms — 139
6.3.1	The gradient descent algorithm — 139
6.3.2	The Landweber algorithm — 141
6.3.3	The conjugate gradient algorithm — 142
6.4	ML-EM algorithms — 143
6.5	OS-EM algorithm — 144
6.6	Noise handling — 144
6.6.1	Analytical methods: windowing — 145
6.6.2	Iterative methods: stopping early — 145
6.6.3	Iterative methods: choosing pixels — 146

6.6.4	Iterative methods: accurate modeling —	148
6.7	Noise modeling as a likelihood function —	149
6.8	Including prior knowledge (Bayesian) —	152
6.9	Mathematical expressions —	153
6.9.1	ART —	153
6.9.2	The Landweber algorithm —	154
6.9.3	CG algorithm —	155
6.9.4	ML-EM —	157
6.9.5	OS-EM —	159
6.9.6	MAP (Green's one-step-late algorithm) —	160
6.9.7	Matched and unmatched projector/backprojector pairs —	162
6.10	Reconstruction using highly undersampled data —	164
6.11	Worked examples —	167
6.12	Summary —	179
	Problems —	180
	Bibliography —	182
7	MRI reconstruction —	185
7.1	The “M” —	185
7.2	The “R” —	187
7.3	The “I” —	190
7.3.1	To obtain z-information: slice selection —	190
7.3.2	To obtain x-information: frequency encoding —	191
7.3.3	To obtain y-information: phase encoding —	192
7.4	Mathematical expressions —	195
7.5	Image reconstruction for MRI —	198
7.5.1	Fourier reconstruction —	198
7.5.2	Iterative reconstruction —	199
7.6	Worked examples —	200
7.7	Summary —	201
	Problems —	202
	Bibliography —	202
8	Using FBP to perform iterative reconstruction —	204
8.1	The Landweber algorithm: from recursive form to nonrecursive form —	204
8.2	The Landweber algorithm: from nonrecursive form to closed form —	205
8.3	The Landweber algorithm: from closed form to backprojection-then-filtering algorithm —	206
8.3.1	Implementation of $(A^T A)^{-1}$ in the Fourier domain —	206

8.3.2	Implementation of $I - (I - \alpha A^T A)^k$ in the Fourier domain —	207
8.3.3	Landweber algorithm: backprojection-then-filtering algorithm —	207
8.3.4	Numerical examples of the window function —	207
8.4	The Landweber algorithm: the weighted FBP algorithm —	209
8.4.1	Landweber algorithm: FBP without noise weighting —	209
8.4.2	Landweber algorithm: FBP with view-based noise weighting —	210
8.4.3	Landweber algorithm: FBP with ray-based noise weighting —	211
8.5	FBP algorithm with quadratic constraints —	213
8.5.1	Example of minimum-norm constrained FBP —	214
8.5.2	Example of reference-image constrained FBP —	216
8.6	Convolution backprojection —	217
8.7	Nonquadratic constraints —	220
8.8	A viewpoint from calculus of variations —	223
8.9	Noise-weighted FBP algorithm for uniformly attenuated SPECT projections —	226
8.10	Other applications of the Landweber-FBP algorithm —	227
8.10.1	The Landweber-FBP version of the iterative FBP algorithm —	227
8.10.2	Estimation of the initial image's contributions to the iterative Landweber reconstruction —	228
8.10.3	FBP implementation of the immediately after-backprojection filtering —	229
8.10.4	Real-time selection of iteration number —	230
8.11	Summary —	230
	Problems —	231
	Bibliography —	231
9	Machine learning —	233
9.1	Analytic algorithms versus iterative algorithms versus machine learning algorithms —	233
9.2	Basic principles of machine learning —	234
9.2.1	Adaptive linear neuron (ADALINE) —	235
9.2.2	Universal approximation —	236
9.2.3	A deep model for a continuous univariate function —	242
9.2.4	When the input is an n -tuple —	245
9.2.5	An n -tuple to m -tuple mapping —	246
9.3	Image reconstruction as an n -tuple to m -tuple mapping —	249
9.4	Convolutional neural network (CNN) as a matched filter for identification —	250
9.5	Convolutional neural network (CNN) as a nonlinear denoise filter —	255
9.6	Super-resolution —	258
9.7	Training —	260

XIV — Contents

9.8	Attention —	261
9.9	Perspectives —	262
9.10	Summary —	265
	Problem —	266
	Bibliography —	267

Index — 269