

Contents

1	Outline	1
	References	3
2	Introduction and Essential Background	5
2.1	Synchrony, Clusters, Chimeras and Coexistence Patterns	5
2.2	The Photoelectrochemical Experiment Behind Our Model	8
2.3	The Stuart-Landau Oscillator, GCLE and MCGLE	11
2.4	Basic Symmetry Considerations	17
2.5	Numerical Methods	22
	References	25
3	From Two-Cluster State to Chimera	31
3.1	Background	32
3.1.1	What is a Chimera State?	32
3.1.2	Origins of Chimeras in Coupled Multi-Component Oscillators	42
3.1.3	Chimeras with Nonlinear Global Coupling	50
3.2	Emergence of Two Minimal Type-II Chimeras	56
3.2.1	A Symmetry-Breaking and a Hopf Bifurcation	58
3.2.2	Period-Doubling Cascade to Non-Symmetric Chimera	71
3.2.3	Symmetry-Increasing Bifurcation Between the Chimera States	76
3.3	Type-II Chimera States with More Oscillators	80
3.3.1	Simply Scaling Up the Minimal Chimera Does Not Work for All N	80
3.3.2	Chimera Developing from an Unbalanced Two-Cluster	89
3.3.3	Cluster Growth of Chimera States for Large Simulation Times	97
3.3.4	Less Symmetric Chimera and Partial Symmetry Increase	104

3.4	Extension to a Spatially Extended System	107
3.5	Chapter Summary, Conclusion and Further Reflections	121
	References	130
4	Coexistence Patterns of Four Oscillators	139
4.1	Background	140
4.1.1	Two Stuart-Landau Oscillators with Nonlinear Global Coupling	141
4.1.2	Linearly Coupled Stuart-Landau Oscillators	144
4.1.3	$N = 4$ Globally Coupled 3D Lasers	147
4.2	Emergence of Different Modulated-Amplitude Solutions	151
4.3	Main Solutions Emerging from $2-2$	160
4.3.1	Outcomes of the Period-Doubling Bifurcation	162
4.3.2	Additional Pitchfork Bifurcations	167
4.3.3	Inner Period-Doubling Bifurcations	169
4.3.4	Torus Bifurcations and Beyond	175
4.3.5	Saddle-Node and Unstable $\mathbb{Z}_4$ Solution	178
4.4	Main Solutions Emerging from $3-1$	181
4.4.1	Transition Region Between the $\mathbb{Z}_2$ and the $\mathbb{S}_2$ Solution	185
4.4.2	Other Instabilities of the $\mathbb{Z}_2$ and $\mathbb{S}_2$ Solutions	189
4.5	Possible Origin of the $\mathbb{Z}_3$ Solutions	193
4.6	Symmetry Transitions of Other Complex States	195
4.7	Chapter Summary, Conclusion and Further Reflections	201
	References	210
5	A Hierarchy of Solutions for $N = 2^n$	213
5.1	Background	214
5.1.1	Milnor Attractors	214
5.1.2	Chaotic Itinerancy and Attractor Ruins	216
5.2	Type-II Solutions for $N = 8$	221
5.2.1	Solutions Emerging Directly from $4-4$	223
5.2.2	More Intricate Coexistence Patterns	230
5.3	Solutions Emerging from $8-8$ for $N = 16$	235
5.3.1	Dynamics Derived from the $8-4-4$ Solution	241
5.3.2	Three-Frequency and Chaotic Dynamics	245
5.4	General Outcomes of Cluster-Splitting Bifurcations	251
5.4.1	Solutions Produced at the Period-Doubling of $N/2-N/2$	253
5.5	Cluster-Halving Cascades for Greater N	256
5.5.1	Two $N = 32$ Examples	256
5.5.2	Further Cluster-Splittings for $N = 64$, $N = 128$ and $N = 256$	264
5.6	A Complex Hierarchy of Cluster Solutions	273
5.6.1	Special Case of Two Equal Smallest Clusters	273
5.6.2	Glimpses of the Multitude of Other Multi-Cluster States	277

5.7	Partial Re-convergence and Attractor Ruins	285
5.7.1	Attractor Ruins in an $\eta = 0.63$ Simulation for $N = 32$	286
5.7.2	Solutions with no Persistent Clusters	293
5.8	Chapter Summary, Conclusion and Further Reflections	298
	References	308
6	Conclusion and Outlook	311
	References	318
Appendix: Equations in the Co-Rotating Frame of the Ensemble		
	Average	319