

Contents

1	From Trajectories to Ensembles in Classical Mechanics	1
1.1	Introduction	1
1.2	Fundamental Grounds of Classical Mechanics.	3
1.2.1	Hamilton's Principle and Equations of Motion	3
1.2.2	Classical Dynamics in Complex Phase Space	10
1.3	From Regular to Chaotic Dynamics	14
1.3.1	Deterministic Chaos and Intrinsic Stochasticity	15
1.3.2	Random Walks and Lévy Flights	21
1.4	Classical Particle Ensembles	23
1.4.1	Scattering Singularities	23
1.4.2	Liouvillian Dynamics	24
1.4.3	Classical Statistical Mechanics as a Field Theory	29
1.4.4	Superposition of Classical Distribution Densities	31
1.5	Classical Mechanics of Continuum Media	33
1.5.1	Coupled Motion	33
1.5.2	Superposition of Classical Waves	36
1.5.3	The Hydrodynamic Approach	39
	References	42
2	Dynamics of Open Classical Systems	47
2.1	Introduction	47
2.2	Dissipative Dynamics	50
2.2.1	Effective Hamiltonians. The Caldirola–Kanai Model	51
2.2.2	Lagrangians for Dissipative Systems and Diffusion Equations	53
2.2.3	The Many-Body Problem	55
2.3	Stochastic Dynamics	57
2.3.1	Brownian Motion and the Langevin Equation	58
2.3.2	Brownian Motion and the Liouville Equation	63
2.3.3	The System-plus-Bath Approach	67

2.4	The Stochastic Hamilton–Jacobi Equation	69
	References	71
3	Elements of Quantum Mechanics	75
3.1	Introduction	75
3.2	Fundamentals of Wave Mechanics.	76
3.2.1	Hamiltonian Analogy and Calculus of Variations	76
3.2.2	Waves and Uncertainty	82
3.2.3	Eigenvalues, Probabilities and Time-Evolution	84
3.2.4	Probability Current Densities and Tunneling	90
3.3	Ensemble Distributions and the Density Matrix.	94
3.3.1	The Wigner Distribution	95
3.3.2	The Husimi Distribution	99
3.4	Feynman’s Path Integrals	102
3.5	The Semiclassical Route to Quantum Mechanics.	105
3.5.1	The Ehrenfest Theorem	105
3.5.2	The JWKB Approximation.	108
3.5.3	The Eikonal Approach.	114
	References	116
4	Optics and Quantum Mechanics.	121
4.1	Introduction	121
4.2	Maxwell’s Equations and the Wave Equation	122
4.3	Interference and Diffraction	124
4.3.1	The Huygens–Fresnel Principle.	124
4.3.2	Interference Phenomena.	128
4.3.3	Young’s Two-Slit Experiment	130
4.3.4	Fresnel and Fraunhofer Diffraction	131
4.3.5	Diffraction by Gratings	136
4.4	Quantization in Bound Optical Systems: Waveguides	138
4.5	Evanescent Waves and Optical Tunneling	142
4.6	Wave Optics and Schrödinger Equation	145
4.6.1	Schrödinger-like Formulation of Electromagnetism	145
4.6.2	Paraxial Approximation and Schrödinger Equation	148
	References	149
5	Dynamics of Open Quantum Systems	153
5.1	Introduction	153
5.2	The Quantization Problem: Standard Theoretical Approaches	156
5.2.1	Time-Dependent (Effective) Hamiltonians	158
5.2.2	Nonlinear Hamiltonians	159
5.2.3	The System-plus-Environment Hamiltonian	162
5.3	Conservative Approach to Dissipative and Stochastic Dynamics	163

5.3.1	The Langevin Formalism	165
5.3.2	Path Integral Formulation: Propagators	177
5.3.3	Markovian Master Equations: The Linblad Equation	179
5.3.4	Stochastic Approaches: Quantum Trajectories	181
	References	182
6	Quantum Mechanics with Trajectories	187
6.1	Introduction	187
6.2	Bohmian Mechanics.	189
6.2.1	Fundamentals	189
6.2.2	Expectation Values and Ensemble Averages.	194
6.2.3	Quantum Hydrodynamics.	198
6.2.4	Bohmian Mechanics in Complex Space	200
6.2.5	Feynman's Paths and Bohmian Trajectories	206
6.3	Towards the Classical Limit in Bohmian Mechanics	207
6.3.1	The JWKB Approximation.	208
6.3.2	Interaction and Entanglement	211
6.3.3	Mixed Bohmian-Classical Mechanics	214
6.3.4	Reduced Quantum Trajectories	217
6.4	Extended Madelung Formulation for Dissipative Systems.	218
6.5	Quantum Stochastic Trajectories. The Beable Interpretation	219
6.6	Are Bohmian Trajectories Real Particle Trajectories?	221
	References	222
7	Trajectories in Optics	231
7.1	Introduction	231
7.2	Geometric Optics: The Optics of Rays.	233
7.2.1	Fermat's Principle.	237
7.2.2	Hamiltonian Analogy and Optical Paths.	238
7.3	The JWKB Approximation	240
7.4	Optical Singularities: Caustics and Rainbows	242
7.5	Hydrodynamical Trajectories in Optics: Photon Paths	243
7.5.1	Polarized Plane Waves	248
7.5.2	Young's Two-Slit Experiment	250
7.5.3	Two-Slit Diffraction and Optical Erasure.	255
7.6	Photon Paths and Optical Schrödinger-like Equations	259
	References	259
	Appendix A: Calculus of Variations.	265
	Appendix B: Stochastic Processes.	269
	Index	295