

Contents

1	Introduction and First Examples	1
1.1	Applications	4
1.2	Organization of the Book	6
1.3	Notes and References	7
 Part I Finite Horizon Optimization Problems and Financial Markets		
2	Theory of Finite Horizon Markov Decision Processes	13
2.1	Markov Decision Models	14
2.2	Finite Horizon Markov Decision Models	17
2.3	The Bellman Equation	19
2.4	Structured Markov Decision Models	28
2.4.1	Semicontinuous Markov Decision Models	29
2.4.2	Continuous Markov Decision Models	32
2.4.3	Measurable Markov Decision Models	33
2.4.4	Monotone and Convex Markov Decision Models	34
2.4.5	Comparison of Markov Decision Models	38
2.5	Stationary Markov Decision Models	39
2.6	Applications and Examples	44
2.6.1	Red-and-Black Card Game	44
2.6.2	A Cash Balance Problem	46
2.6.3	Stochastic Linear-Quadratic Problems	50
2.7	Exercises	53
2.8	Remarks and References	56
3	The Financial Markets	59
3.1	Asset Dynamics and Portfolio Strategies	59
3.2	Jump Markets in Continuous Time	66
3.3	Weak Convergence of Financial Markets	69
3.4	Utility Functions and Expected Utility	70

3.5	Exercises	72
3.6	Remarks and References	73
4	Financial Optimization Problems	75
4.1	The One-Period Optimization Problem	76
4.2	Terminal Wealth Problems	79
4.3	Consumption and Investment Problems	93
4.4	Optimization Problems with Regime Switching	100
4.5	Portfolio Selection with Transaction Costs	106
4.6	Dynamic Mean-Variance Problems	117
4.7	Dynamic Mean-Risk Problems	124
4.8	Index-Tracking	132
4.9	Indifference Pricing	134
4.10	Approximation of Continuous-Time Models	140
4.11	Remarks and References	142

Part II Partially Observable Markov Decision Problems

5	Partially Observable Markov Decision Processes	147
5.1	Partially Observable Markov Decision Processes	148
5.2	Filter Equations	151
5.3	Reformulation as a Standard Markov Decision Model	157
5.4	Bayesian Decision Models	159
5.5	Bandit Problems with Finite Horizon	166
5.6	Exercises	171
5.7	Remarks and References	173
6	Partially Observable Markov Decision Problems in Finance	175
6.1	Terminal Wealth Problems	176
6.2	Dynamic Mean-Variance Problems	183
6.3	Remarks and References	188

Part III Infinite Horizon Optimization Problems

7	Theory of Infinite Horizon Markov Decision Processes	193
7.1	Markov Decision Models with Infinite Horizon	194
7.2	Semicontinuous Markov Decision Models	201
7.3	Contracting Markov Decision Models	205
7.4	Positive Markov Decision Models	208
7.5	Computational Aspects	211
7.5.1	Howard's Policy Improvement Algorithm	212
7.5.2	Linear Programming	214
7.5.3	State Space Discretization	220
7.6	Applications and Examples	223
7.6.1	Markov Decision Models with Random Horizon	223
7.6.2	A Cash Balance Problem with Infinite Horizon	224

7.6.3	Casino Games	226
7.6.4	Bandit Problems with Infinite Horizon	230
7.7	Exercises	236
7.8	Remarks and References	240
8	Piecewise Deterministic Markov Decision Processes	243
8.1	Piecewise Deterministic Markov Decision Models	243
8.2	Solution via a Discrete-Time Markov Decision Process	247
8.3	Continuous-Time Markov Decision Chains	256
8.4	Exercises	262
8.5	Remarks and References	264
9	Optimization Problems in Finance and Insurance	267
9.1	Consumption-Investment Problems with Random Horizon ...	267
9.2	A Dividend Problem in Risk Theory	271
9.3	Terminal Wealth Problems in a Pure Jump Market	280
9.4	Trade Execution in Illiquid Markets	293
9.5	Remarks and References	298

Part IV Stopping Problems

10	Theory of Optimal Stopping Problems	303
10.1	Stopping Problems with Finite Horizon	303
10.2	Stopping Problems with Unbounded Horizon	309
10.3	Applications and Examples	316
10.3.1	A House Selling Problem	316
10.3.2	Quiz Show	318
10.3.3	The Secretary Problem	319
10.3.4	A Bayesian Stopping Problem	323
10.4	Exercises	329
10.5	Remarks and References	330
11	Stopping Problems in Finance	331
11.1	Pricing of American Options	331
11.2	Credit Granting	340
11.3	Remarks and References	343

Part V Appendix

A	Tools from Analysis	347
A.1	Semicontinuous Functions	347
A.2	Set-Valued Mappings and a Selection Theorem	351
A.3	Miscellaneous	352

B	Tools from Probability	355
	B.1 Probability Theory	355
	B.2 Stochastic Processes	356
	B.3 Stochastic Orders	358
C	Tools from Mathematical Finance	365
	C.1 No Arbitrage Pricing Theory	365
	C.2 Risk Measures	367
	References	369
	Index	385