Contents

Preface — V

Acknowledgment — XIII

1 Spectral flow in finite dimension — 1
1.1 From intuition to definition — 1
1.2 Structural properties of the spectral flow — 4
1.3 Alternative expressions for the spectral flow — 5
1.4 Spectral flow as a sum of eigenvalue crossings — 8
1.5 The spectral flow for paths of unitaries — 11
1.6 The spectral flow through the imaginary axis — 19
1.7 $\mathbb{Z}_2$-valued orientation flow of skew-adjoint real matrices — 20
1.8 Multiparameter spectral flow — 23

2 Applications of finite-dimensional spectral flow — 25
2.1 Bott–Maslov index in finite dimension — 25
2.2 J-unitary matrices — 31
2.3 Conley–Zehnder index in finite dimension — 39
2.4 Oscillation theory for finite Jacobi matrices — 48
2.5 Oscillation theory for periodic Jacobi matrices — 54
2.6 Bound states for scattering systems — 56

3 Bounded Fredholm operators — 64
3.1 Compact operators and their spectral theory — 64
3.2 Basic properties of bounded Fredholm operators — 66
3.3 The index of a Fredholm operator — 69
3.4 The notion of essential spectrum — 75
3.5 Spectral flow in finite dimension as index — 78
3.6 Bounded self-adjoint Fredholm operators — 83
3.7 Essentially gapped unitary operators — 87

4 Spectral flow for bounded self-adjoint Fredholm operators — 89
4.1 The definition of the spectral flow — 89
4.2 Fundamental properties of the spectral flow — 92
4.3 Formulas for the spectral flow — 95
4.4 Spectral flow for essentially hyperbolic operators — 107
4.5 Spectral flow for paths of essentially gapped unitaries — 109
4.6 Connecting spectral flows of self-adjoints and unitaries — 116

Bibliografische Informationen
https://d-nb.info/1283839326

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

5 Fredholm pairs and their index — 126
5.1 Projections and orthogonal projections — **126**
5.2 Characterization of Fredholm pairs of projections — **130**
5.3 Fredholm pairs of orthogonal projections — **137**
5.4 Fredholm pairs of symmetries — **150**
5.5 Fredholm pairs of unitary conjugate projections — **152**
5.6 Spectral flow of linear paths between Fredholm pairs — **156**
5.7 Spectral flow formulas for paths with compact difference — **158**
5.8 Spectral flow as sum of indices of Fredholm pairs — **165**
5.9 Relative Morse indices and spectral flow — **167**

6 Unbounded Fredholm operators — 173
6.1 Topologies on closed and densely defined operators — **173**
6.2 Basic properties of unbounded Fredholm operators — **183**
6.3 Unbounded self-adjoint Fredholm operators — **195**
6.4 Self-adjoint Fredholm operators with compact resolvent — **203**

7 Spectral flow for unbounded self-adjoint Fredholm operators — 218
7.1 Definition of spectral flow and its basic properties — **218**
7.2 The η-invariant and spectral flow — **223**
7.3 Conley–Zehnder index as spectral flow — **229**
7.4 Spectral flow as index via semiclassics — **233**

8 Homotopy theory of Fredholm operators — 247
8.1 Homotopy groups of essentially gapped unitaries — **247**
8.2 Homotopy groups of Fredholm operators — **248**
8.3 Homotopy groups of bounded self-adjoint Fredholm operators — **251**
8.4 An application: characterization of spectral flow — **255**
8.5 Homotopy groups of Fredholm pairs — **258**
8.6 Homotopy groups of unbounded self-adjoint Fredholm operators — **268**
8.7 Resumé: homotopy equivalences of operator classes — **275**

9 Bott–Maslov index via spectral flow — 277
9.1 Krein spaces and operators thereon — **277**
9.2 J-isotropic subspaces — **281**
9.3 J-Lagrangian subspaces — **287**
9.4 Fredholm pairs of J-Lagrangian projections — **291**
9.5 Paths of Fredholm pairs of J-Lagrangian projections — **296**
9.6 Conley–Zehnder index — **303**
9.7 Oscillation theory for bound states of scattering systems — **308**

10 Index pairings and spectral localizer — 313
10.1 Fredholm modules and index pairings — **313**
10.2 Spectral flow formulas for index pairings — **319**
10.3 Spectral localizer for even index pairings — **320**
10.4 Spectral localizer for odd index pairings — **329**
10.5 The η-invariant of the spectral localizer — **335**

11 Spectral flow in semifinite von Neumann algebras — 345
11.1 Fredholm operators in semifinite von Neumann algebras — **345**
11.2 $(P \cdot Q)$-Fredholm operators — **358**
11.3 Semifinite Fredholm pairs of projections — **365**
11.4 Definition and basic properties of the spectral flow — **370**
11.5 Index formulas for semifinite spectral flow — **375**
11.6 Semifinite spectral localizer — **379**

12 Spectral flow in bifurcation theory — 389
12.1 A primer of variational bifurcation theory — **389**
12.2 The spectral flow in variational bifurcation theory — **394**
12.3 Applications to Hamiltonian systems — **404**

A Collection of technical elements — 417
A.1 Riesz projections — **417**
A.2 Norm estimates on roots — **419**
A.3 Definitions and facts from topology — **420**

Acronyms and notations — 427

Bibliography — 431

Index — 439