

Contents

Introduction	XI
--------------------	----

Part I. Advanced Replica-Symmetry

8. The Gardner Formula for the Sphere	3
8.1 Introduction	3
8.2 Gaussian Processes	4
8.3 The Gardner Formula for the Gaussian Measure	13
8.4 The Gardner Formula for the Sphere	24
8.5 The Bernoulli Model	34
9. The Gardner Formula for the Discrete Cube	49
9.1 Overview	49
9.2 A Priori Estimates	52
9.3 Gaussian Processes	58
9.4 Integration by Parts	67
9.5 The Replica Symmetric Solution	72
9.6 The Gardner Formula for the Discrete Cube	83
9.7 Higher Order Expansion and Central Limit Theorems	90
9.8 An Approximation Procedure	109
9.9 The Bernoulli Model	115
10. The Hopfield Model	123
10.1 Introduction	123
10.2 The Replica-Symmetric Equations	124
10.3 Localization on Balls with Random Centers	126
10.4 Controlling $m_k(\sigma)$, $k \geq 2$	141
10.5 The Smart Path	145
10.6 Integration by Parts	157
10.7 The Replica-Symmetric Solution	162
10.8 Computing $p_{N,M}$	174
10.9 Higher Moments, the TAP Equations	179
10.10 Central Limit Theorems	191

10.11 The p -Spin Hopfield Model.....	198
10.12 Proof of Theorem 10.2.1	216
11. The SK Model Without External Field	225
11.1 Overview	225
11.2 Lower Deviations for Z_N	226
11.3 Upper Deviations for Z_N	233
11.4 The Aizenman-Lebowitz-Ruelle Central Limit Theorem.....	239
11.5 The Matrix of Spin Correlations	243
11.6 The Model with d -Component Spins	253
11.7 A Research Problem: The Transition at $\beta = 1$	268
Part II. Low Temperature	
12. The Ghirlanda-Guerra Identities.....	287
12.1 The Identities	287
12.2 The Extended Identities	296
12.3 A Positivity Principle	298
12.4 The Distribution of the Overlaps at Given Disorder	303
12.5 Large Deviations	307
13. The High-Temperature Region of the SK Model.....	313
13.1 The Poisson-Dirichlet Distribution and the REM	313
13.2 The 1-RSB Bound for the SK Model	319
13.3 Toninelli's Theorem	323
13.4 Overview of Proof	326
13.5 A Bound for Coupled Copies	331
13.6 The Main Estimate	335
13.7 Exponential Inequalities	343
14. The Parisi Formula	349
14.1 Introduction	349
14.2 Poisson-Dirichlet Cascades	349
14.3 Fundamental Identities	353
14.4 Guerra's Broken Replica-Symmetry Bound	362
14.5 Method of Proof	371
14.6 Bounds for Coupled Copies	381
14.7 Operators.....	395
14.8 Main Estimate: Methodology	415
14.9 Main Estimate: The Critical Cases	419
14.10 Main Estimate: Proof of Proposition 14.8.6	428
14.11 Parisi Measures.....	439
14.12 Positivity of the Overlap.....	448
14.13 Notes and Comments	473

15. The Parisi Solution	475
15.1 Introduction	475
15.2 Ghirlanda-Guerra Identities and Poisson Dirichlet Cascades	475
15.3 The Baffioni-Rosati Theorem	483
15.4 Generic Sequences and Pure States	491
15.5 Determinators; Panchenko's Invariance Theorem	500
15.6 Panchenko's Ultrametricity Theorem	517
15.7 Problems: Strong Ultrametricity and Chaos	528
15.8 The Aizenman-Sims-Starr Scheme	538
15.9 Probability Measures on Hilbert Space	548
15.10 Notes and Comments	558
16. The p-Spin Interaction Model	559
16.1 Overview	559
16.2 Poisson-Dirichlet Distribution and Ghirlanda-Guerra Identities	560
16.3 A Priori Estimates	564
16.4 The Lumps and Their Weights	574
16.5 One Step of Replica-Symmetry Breaking	579
16.6 Computing $p_N(\beta)$	592
16.7 A Research Problem: The Dynamical Transition	601
16.8 Notes and Comments	602
A. Appendix: Elements of Probability Theory	603
A.1 How to Use This Appendix	603
A.2 Gaussian Random Variables	603
A.3 Gaussian Integration by Parts	606
A.4 Tail Estimates	607
A.5 How to Use Tail Estimates	610
A.6 Bernstein's Inequality	612
A.7 ε -Nets	614
A.8 Random Matrices	615
A.9 Poisson Random Variables and Point Processes	620
A.10 The Paley-Zygmund Inequality	622
A.11 Differential Inequalities	622
References	623
Index	629