

# Contents

Introduction . . . . .	ix
General Notations and Numbering . . . . .	xii
CHAPTER I	
Classical Theory of Vibration for Systems with Infinitely Many Degrees of Freedom . . . . .	1
1. Introduction . . . . .	1
2. Elements of Vibration Theory for Systems with $n$ Degrees of Freedom . . . . .	2
3. Infinite-Dimensional Separable Hilbert Spaces . . . . .	5
4. A Class of Compact Self-Adjoint Operators . . . . .	9
5. Introduction of the Spaces $V$ and $H$ Associated with the Elastic and Kinetic Energies . . . . .	15
6. The Standard Vibration Problem for a System with Discrete Spectrum . . . . .	21
7. Variational Properties of Eigenvalues. Rayleigh Principle. Minimax Principle and Comparison Theorem . . . . .	24
CHAPTER II	
Some Classical Vibration Problems . . . . .	27
1. Introduction . . . . .	27
2. Distributions and Sobolev Spaces . . . . .	28
3. Vibrating Membrane . . . . .	37
4. Examples and Remarks about Strings and Membranes. A Form of the Saint-Venant Principle . . . . .	42
5. Linear Shallow-Water Oscillations. Neumann Boundary Condition . . . . .	45
6. Complement. A Problem without Boundary Conditions . . . . .	50
7. Vibration of a Three-Dimensional Elastic Body. Application of the Comparison Theorem and Particular Cases . . . . .	53
8. Small Oscillations of a Compressible Fluid in a Vessel with or without Free Surface . . . . .	61
9. Exercises . . . . .	68
CHAPTER III	
Elements of Operator Theory . . . . .	70
1. Generalities on Banach Spaces and Operators . . . . .	70

2. Unbounded Linear Operators. Closed Operators . . . . .	74
3. Resolvents and Spectra . . . . .	80
4. Singularities of the Resolvent. Fredholm Alternative . . . . .	82
5. Spectra of Compact and Anticompact Operators . . . . .	88
6. Symmetric and Self-Adjoint Operators . . . . .	91
7. Spectral Families . . . . .	95
8. Semigroups . . . . .	101
9. Some General Remarks on the Regularity Theory for Elliptic Equations and the System of Elasticity . . . . .	109
10. Trace Theorems for Solutions of Elliptic Equations. The Elements of the Lions–Magenes Theory . . . . .	117
11. Comments and Exercises . . . . .	122
 CHAPTER IV	
Examples of Nonstandard Vibrations and Coupling . . . . .	127
1. The Thermoelasticity System . . . . .	127
2. Vibration of a Viscoelastic Solid . . . . .	133
3. Essential Spectrum. First Example of a Vibrating System without Compactness . . . . .	139
4. An Example of Compact–Noncompact Coupled Vibrating System . . . . .	143
5. Bloch Waves and Related Topics . . . . .	148
6. Systems Containing a Part without Kinetic Energy . . . . .	154
7. Plates—Coupling of Flexion and Traction Modes . . . . .	158
8. A Problem where the Part without Kinetic Energy Is Unbounded . . . . .	161
9. Comments and Problems . . . . .	166
 CHAPTER V	
Spectral Perturbation . . . . .	170
1. Generalities. The Implicit Function Theorem, the Weierstrass Preparation Theorem, and Holomorphic Functions with Values in a Banach Space . . . . .	170
2. Eigenvalues of Matrices Depending Holomorphically on a Parameter . . . . .	174
3. Power Series Expansions for Eigenvalues and Eigenvectors . . . . .	178
4. Spectral Perturbations for Anticompact Operators Associated with a Holomorphic Sesquilinear Family . . . . .	184
5. Complements and Generalizations . . . . .	186
6. First Example: Smooth Perturbation of the Boundary . . . . .	192
7. Some Implicit Holomorphic Eigenvalue Problems . . . . .	194
8. Perturbation of an Eigenvalue of Multiplicity Two of a Self-Adjoint Operator Depending on Two Parameters $z_1, z_2$ . . . . .	199
9. Eigenvalue Problem for Families Depending Nonanalytically on a Parameter . . . . .	202
10. Some Implicit Nonholomorphic Eigenvalue Problems . . . . .	207
11. Perturbation of Spectral Families. Rellich’s Theorem . . . . .	210
12. Remarks on Time-Dependent Solutions of Standard Vibration Problems . . . . .	213
13. Numerical Computation of Spectral Families . . . . .	215
14. Complements and Problems . . . . .	218

<b>CHAPTER VI</b>	
<b>Formal Perturbation Methods</b>	221
1. Introduction	221
2. The Order Symbols $o$ and $O$ . Gauge Functions	222
3. Singular Perturbation. Asymptotic Expansion of the Explicit Solution for a Model Boundary Value Problem	225
4. Asymptotic Study of the Solution to the Model Problem from the Equation and the Boundary Conditions	228
5. Comments and Heuristic Ideas for Other Problems	232
6. Matching Rule of Kaplun and Lagerstrom	232
7. An Interpretation of the Matching	234
8. Matching by Intermediate Variables	236
9. Extension Theorem of Kaplun	238
10. Introduction to Two-Scale Problems. Linear Oscillator with Small Damping	239
11. Second Example. Van der Pol Oscillator	242
12. Van der Pol's Transformation and Average Method	244
13. Integral Continuity. Error Estimate for the Average and Two-Scale Methods	245
14. Moment Expansion of a Function with Shrinking Support	250
15. Exercises	252
<b>CHAPTER VII</b>	
<b>Perturbation of Vibrating Systems</b>	254
1. A Model Stiff Problem. Expansions for Eigenvalues and Eigenvectors	254
2. Justification of the Preceding Expansions	261
3. Elastic Body Coupled with a Gas of Small Density (Bounded Domains)	265
4. Vibration of an Almost Incompressible Elastic Body	269
5. Spectral Families in Large Domains. Application to High Frequency Homogenization	274
6. A New Class of Stiff Problems. Low and High Frequencies	277
7. Plate with Small Rigidity	285
8. Vibrations of a Slightly Viscous Gas	292
9. Thermoelastic Body with Small Thermal Conductivity	298
10. General Considerations on Vibrations of Systems with Concentrated Masses	304
11. Concentrated Masses. Local Vibrations in the Case $N = 3, m > 2$	308
12. Concentrated Masses. Global Vibrations for Space Dimension $N = 2$ or $3$	312
13. Concentrated Masses. Global Vibrations for Space Dimension $N = 1$ and $m = 1$	315
14. Comments and Problems	320
<b>CHAPTER VIII</b>	
<b>The Helmholtz Equation in Unbounded Domains</b>	322
1. Generalities on the Helmholtz Equation in a Neighborhood of Infinity. Radiation Condition	322
2. Some Properties of the Wave Equation in the Space-Time	335

3. Existence, Uniqueness, and Scattering Frequencies for the Dirichlet Problem in an Outer Domain with Smooth Boundary . . . . .	344
4. Dirichlet, Neumann, and Transmission Problems in an Outer Domain with not Necessarily Smooth Boundary. Examples and Complements . . . . .	350
5. Spectrum and Spectral Family of the Laplacian in an Outer Domain. Limiting Absorption . . . . .	358
6. Local Decay of Solutions as $t \rightarrow \infty$ . . . . .	362
7. Limiting Amplitude . . . . .	365
8. The Rudiments of the Lax and Phillips Theory of Scattering . . . . .	370
9. Comments and Exercises . . . . .	373
 <b>CHAPTER IX</b>	
<b>Scattering Problems Depending on a Parameter. Elastic Structure–Fluid Interaction in Unbounded Domains . . . . .</b>	<b>376</b>
1. Introduction . . . . .	376
2. Elastic Body Surrounded by a Compressible Fluid . . . . .	378
3. Asymptotics for a Fluid of Small Density. Resonance of Two Bodies Across Air . . . . .	384
4. Asymptotics for a Fluid with Small Compressibility. Low and High Frequencies . . . . .	389
5. The Helmholtz Resonator. Asymptotic Expansion for the Solution . . . . .	394
6. Complements and Problems . . . . .	401
 <b>References . . . . .</b>	<b>403</b>
 <b>Index . . . . .</b>	<b>417</b>