

Contents

1	Preliminaries	1
1.1	Finite Spaces and Posets	2
1.2	Maps, Homotopies and Connectedness.....	4
1.3	Homotopy Types.....	6
1.4	Weak Homotopy Types: The Theory of McCord.....	10
2	Basic Topological Properties of Finite Spaces	19
2.1	Homotopy and Contiguity	19
2.2	Minimal Pairs	20
2.3	T_1 -Spaces	22
2.4	Loops in the Hasse Diagram and the Fundamental Group	22
2.5	Euler Characteristic	25
2.6	Automorphism Groups of Finite Posets	27
2.7	Joins, Products, Quotients and Wedges.....	29
2.8	A Finite Analogue of the Mapping Cylinder	34
3	Minimal Finite Models	37
3.1	A Finite Space Approximation.....	37
3.2	Minimal Finite Models of the Spheres	39
3.3	Minimal Finite Models of Graphs	40
3.4	The $f^\infty(X)$	44
4	Simple Homotopy Types and Finite Spaces	49
4.1	Whitehead's Simple Homotopy Types.....	50
4.2	Simple Homotopy Types: The First Main Theorem.....	53
4.3	Joins, Products, Wedges and Collapsibility	60
4.4	Simple Homotopy Equivalences: The Second Main Theorem	64
4.5	A Simple Homotopy Version of Quillen's Theorem A.....	68
4.6	Simple, Strong and Weak Homotopy in Two Steps	70

5	Strong Homotopy Types	73
5.1	A Simplicial Notion of Homotopy	73
5.2	Relationship with Finite Spaces and Barycentric Subdivisions	77
5.3	Nerves of Covers and the Nerve of a Complex	80
6	Methods of Reduction	85
6.1	Osaki's Reduction Methods	85
6.2	γ -Points and One-Point Reduction Methods	87
7	h-Regular Complexes and Quotients	93
7.1	h-Regular CW-Complexes and Their Associated Finite Spaces	93
7.2	Quotients of Finite Spaces: An Exact Sequence for Homology Groups	100
8	Group Actions and a Conjecture of Quillen	105
8.1	Equivariant Homotopy Theory for Finite Spaces	106
8.2	The Poset of Nontrivial p -Subgroups and the Conjecture of Quillen	108
8.3	Equivariant Simple Homotopy Types	111
8.4	Applications to Quillen's Work	119
9	Reduced Lattices	121
9.1	The Homotopy of Reduced Lattices	121
9.2	The $\mathcal{L}$ Complex	124
10	Fixed Points and the Lefschetz Number	129
10.1	The Fixed Point Property for Finite Spaces	129
10.2	On the Lefschetz Theorem for Simplicial Automorphisms	133
11	The Andrews–Curtis Conjecture	137
11.1	n -Deformations and Statements of the Conjectures	137
11.2	Quasi Constructible Complexes	139
11.3	The Dual Notion of Quasi Constructibility	145
12	Appendix	151
A.1	Simplicial Complexes	151
A.2	CW-Complexes and a Gluing Theorem	155
	References	161
	List of Symbols	165
	Index	167