

Table of Contents

| | |
|--------------------------------------|-------------|
| Foreword | v |
| Preface to the Second Edition | xiii |
| Preface to the First Edition | xxi |

| | | |
|----------|--|----------|
| I | Some Results from Scattering Theory | 1 |
|----------|--|----------|

| | | |
|-----|--|----|
| I.1 | The Reduced Radial Schrödinger Equation | 1 |
| I.2 | The Regular Solution: S-Wave ($l = 0$) | 7 |
| I.3 | The Jost Solution: S-Wave ($l = 0$) | 9 |
| I.4 | The Jost Function and the Phase Shift | 11 |
| I.5 | Higher Waves | 13 |
| I.6 | Singular Potentials | 16 |
| I.7 | Comments and References | 19 |

| | | |
|-----------|--|-----------|
| II | Bound States—Eigenfunction Expansions | 20 |
|-----------|--|-----------|

| | | |
|------|---|----|
| II.1 | Bound States: The Levinson Theorem | 20 |
| II.2 | Integral Representation for the Jost Function | 25 |
| II.3 | Eigenfunction Expansion | 26 |
| II.4 | Miscellaneous Results | 29 |
| II.5 | Singular Potentials | 34 |
| II.6 | Comments and References | 38 |

| | | |
|------------|--|-----------|
| III | The Gel'fand–Levitan–Jost–Kohn Method | 40 |
|------------|--|-----------|

| | | |
|-------|--|----|
| III.1 | The Povzner–Levitan Representation | 40 |
| III.2 | The Gel'fand–Levitan Integral Equation | 46 |

| | | |
|-------------|---|------------|
| III.3 | Krein's Equation | 48 |
| III.4 | Higher Waves | 49 |
| III.5 | More General Equations | 51 |
| III.6 | Concluding Summary of the Method | 54 |
| III.7 | Comments and References | 56 |
| IV | Applications of the Gel'fand-Levitan Equation | 59 |
| IV.1 | Introduction of New Bound States | 59 |
| IV.2 | Phase Equivalent Potentials | 64 |
| IV.3 | Bargmann Potentials | 67 |
| IV.4 | Transformations of the Schrödinger Equation | 70 |
| IV.5 | Comments and References | 74 |
| V | The Marchenko Method | 76 |
| V.1 | The Levin Representation | 76 |
| V.2 | The Marchenko Integral Equation | 78 |
| V.3 | Comments and References | 85 |
| VI | Examples | 86 |
| VI.1 | Bargmann Potentials | 86 |
| VI.2 | Singular Potentials | 92 |
| VI.3 | Comments and References | 93 |
| VII | Special Classes of Potentials | 95 |
| VII.1 | Yukawa Potentials and the Direct Problem | 95 |
| VII.2 | Yukawa Potentials and the Inverse Problem | 100 |
| VII.3 | Higher Waves—Coulomb Potential | 106 |
| VII.4 | Holomorphic Potentials | 109 |
| VII.5 | Comments and References | 110 |
| VIII | Nonlocal Separable Interactions | 112 |
| VIII.1 | The Direct Problem | 112 |
| VIII.2 | The Inverse Problem | 118 |
| VIII.3 | More General Interactions | 122 |
| VIII.4 | Applications | 124 |
| VIII.5 | Comments and References | 126 |
| IX | Miscellaneous Approaches to the Inverse Problems at Fixed l | 128 |
| IX.1 | Generalization to Other Central Potentials | 128 |
| IX.2 | Krein's Approach | 131 |
| IX.3 | Systems of Equations | 132 |
| IX.4 | Coupled Channels | 135 |

| | | |
|------|-------------------------------|-----|
| IX.5 | Relativistic Problems | 139 |
| IX.6 | Discrete Forms of the Methods | 144 |
| IX.7 | Dispersion Relation Approach | 147 |
| IX.8 | Energy Dependent Potentials | 151 |
| IX.9 | Miscellaneous Results | 152 |

X Scattering Amplitudes from Elastic Cross Sections 155

| | | |
|-----|--|-----|
| X.1 | Introduction | 155 |
| X.2 | Constructive Methods | 160 |
| X.3 | Other Uniqueness Studies | 164 |
| X.4 | Local Results | 168 |
| X.5 | Uniqueness and Stability: A Reassessment | 173 |
| X.6 | Generalizations | 177 |
| X.7 | Comments and References | 180 |

XI Potentials from the Scattering Amplitude at Fixed Energy: General Equation and Mathematical Tools 182

| | | |
|------|--|-----|
| XI.1 | Introduction | 182 |
| XI.2 | The Transformation Kernel | 185 |
| XI.3 | The Symmetric Kernel and the Integral Equation | 186 |
| XI.4 | The General Machinery | 189 |
| XI.5 | Further Study of the Integral Equation | 191 |
| XI.6 | Remarks on This Chapter | 193 |
| XI.7 | Connection Between the Problem at Fixed E and the Problem at Fixed l | 194 |

XII Potentials from the Scattering Amplitude at Fixed Energy: Matrix Methods 195

| | | |
|-------|---|-----|
| XII.1 | Introduction | 195 |
| XII.2 | A Method in Which the Index μ Runs Through Integers | 196 |
| XII.3 | Inversion of the Matrix M and Other Properties | 197 |
| XII.4 | Construction of $\{c_i\}$ from $\{\tan \delta_i\}$ | 200 |
| XII.5 | Construction of $V(r)$ —Consistency of the Method | 203 |
| XII.6 | Generalized Matrix Methods | 205 |
| XII.7 | Miscellaneous Results | 207 |
| XII.8 | Interpolation Properties | 209 |
| XII.9 | Limitation of the Matrix Methods | 212 |

XIII Potentials from the Scattering Amplitude at Fixed Energy: Operator Methods 214

| | | |
|--------|---|-----|
| XIII.1 | Introduction | 214 |
| XIII.2 | Method for Potentials of the Yukawa Class | 215 |

| | | |
|--------------|---|------------|
| XIII.3 | Methods Using the Spectrum of the Schrödinger Operator | 217 |
| XIII.4 | Complete Solution | 223 |
| XIII.5 | Remarks on the Methods | 233 |
| XIV | The Three-Dimensional Inverse Problem | 235 |
| XIV.1 | Introduction | 235 |
| XIV.2 | New Outline of One-Dimensional Methods | 237 |
| XIV.3 | Approach of the Three-Dimensional Problem Based on Faddeev's Green's Function | 242 |
| XIV.4 | Consistency and $\bar{\partial}$ -Approaches | 251 |
| XIV.5 | Other Approaches in the Frequency Domain | 256 |
| XIV.6 | Time-Domain Inverse Scattering Theory | 265 |
| XIV.7 | Comments and References | 272 |
| XV | Miscellaneous Approaches to Inverse Problems at Fixed Energy | 275 |
| XV.1 | Methods Using Interpolation Properties | 275 |
| XV.2 | Methods Using Generalized Translation Operators | 285 |
| XV.3 | Remark on the Results Given in This Chapter | 289 |
| XVI | Approximate Methods | 290 |
| XVI.1 | Introduction | 290 |
| XVI.2 | Born Approximation | 292 |
| XVI.3 | The Semiclassical Approximation I | 295 |
| XVI.4 | Semiclassical Analysis II | 303 |
| XVI.5 | Semiclassical Analysis III | 310 |
| XVI.6 | From Approximate to Exact Methods | 317 |
| XVI.7 | Semiclassical Studies in Other Fields | 319 |
| XVII | Inverse Problems in One Dimension | 323 |
| XVII.1 | Introduction | 323 |
| XVII.2 | The Inverse Problem: Approaches Related with a Marchenko Equation | 333 |
| XVII.3 | The Inverse Problem: Other Approaches | 343 |
| XVII.4 | More General One-Dimensional Problems | 353 |
| XVII.5 | Comments and References | 378 |
| XVII.A | Appendix (Exercises for Readers) | 382 |
| XVIII | Problems Connected with Discrete Spectra | 389 |
| XVIII.1 | Introduction | 389 |
| XVIII.2 | Relations with Other Problems and Extensions | 402 |
| XVIII.3 | Inverse Problem in the Coupling Constant | 406 |
| XVIII.4 | Comments and References | 413 |

| | | |
|------------|---|------------|
| XIX | Numerical Problem | 416 |
| XIX.1 | Introduction | 416 |
| XIX.2 | Numerical Methods for Local Potentials | 426 |
| XIX.3 | Numerical Methods for Inverse Spectral Problems | 437 |
| XIX.4 | Nonlocal Potentials | 439 |
| | Reference List | 441 |
| | Subject Index | 495 |