

Contents

Preface	vii
Introduction	1

PART I	
The One-Center Point Interaction	9

CHAPTER 1.1	
The One-Center Point Interaction in Three Dimensions	11
1.1.1 Basic Properties	11
1.1.2 Approximations by Means of Local as well as Nonlocal Scaled Short-Range Interactions	17
1.1.3 Convergence of Eigenvalues and Resonances	28
1.1.4 Stationary Scattering Theory	37
Notes	46

CHAPTER 1.2	
Coulomb Plus One-Center Point Interaction in Three Dimensions	52
1.2.1 Basic Properties	52
1.2.2 Approximations by Means of Scaled Coulomb-Type Interactions	57
1.2.3 Stationary Scattering Theory	66
Notes	74

CHAPTER I.3	
The One-Center δ-Interaction in One Dimension	75
I.3.1 Basic Properties	75
I.3.2 Approximations by Means of Local Scaled Short-Range Interactions	79
I.3.3 Convergence of Eigenvalues and Resonances	83
I.3.4 Stationary Scattering Theory	85
Notes	89
 CHAPTER I.4	
The One-Center δ'-Interaction in One Dimension	91
Notes	95
 CHAPTER I.5	
The One-Center Point Interaction in Two Dimensions	97
Notes	105
 PART II	
Point Interactions with a Finite Number of Centers	107
 CHAPTER II.1	
Finitely Many Point Interactions in Three Dimensions	109
II.1.1 Basic Properties	109
II.1.2 Approximations by Means of Local Scaled Short-Range Interactions	121
II.1.3 Convergence of Eigenvalues and Resonances	125
II.1.4 Multiple Well Problems	132
II.1.5 Stationary Scattering Theory	134
Notes	138
 CHAPTER II.2	
Finitely Many δ-Interactions in One Dimension	140
II.2.1 Basic Properties	140
II.2.2 Approximations by Means of Local Scaled Short-Range Interactions	145
II.2.3 Convergence of Eigenvalues and Resonances	148
II.2.4 Stationary Scattering Theory	150
Notes	153
 CHAPTER II.3	
Finitely Many δ'-Interactions in One Dimension	154
Notes	159
 CHAPTER II.4	
Finitely Many Point Interactions in Two Dimensions	160
Notes	165

PART III

Point Interactions with Infinitely Many Centers 167

CHAPTER III.1

Infinitely Many Point Interactions in Three Dimensions 169

III.1.1 Basic Properties 169

III.1.2 Approximations by Means of Local Scaled Short-Range Interactions 173

III.1.3 Periodic Point Interactions 176

III.1.4 Crystals 178

III.1.5 Straight Polymers 200

III.1.6 Monomolecular Layers 210

III.1.7 Bragg Scattering 217

III.1.8 Fermi Surfaces 226

III.1.9 Crystals with Defects and Impurities 239
Notes 250

CHAPTER III.2

Infinitely Many δ -Interactions in One Dimension 253

III.2.1 Basic Properties 253

III.2.2 Approximations by Means of Local Scaled Short-Range Interactions 261

III.2.3 Periodic δ -Interactions 263

III.2.4 Half-Crystals 284

III.2.5 Quasi-periodic δ -Interactions 288III.2.6 Crystals with Defects and Impurity Scattering 290
Notes 303

CHAPTER III.3

Infinitely Many δ' -Interactions in One Dimension 307

Notes 323

CHAPTER III.4

Infinitely Many Point Interactions in Two Dimensions 324

Notes 333

CHAPTER III.5

Random Hamiltonians with Point Interactions 334

III.5.1 Preliminaries 334

III.5.2 Random Point Interactions in Three Dimensions 341

III.5.3 Random Point Interactions in One Dimension 349

Notes 353

APPENDICES

A Self-Adjoint Extensions of Symmetric Operators 357

B Spectral Properties of Hamiltonians Defined as Quadratic Forms 360

C	Schrödinger Operators with Interactions Concentrated Around Infinitely Many Centers	365
D	Boundary Conditions for Schrödinger Operators on $(0, \infty)$	371
E	Time-Dependent Scattering Theory for Point Interactions	374
F	Dirichlet Forms for Point Interactions	376
G	Point Interactions and Scales of Hilbert Spaces	380
H	Nonstandard Analysis and Point Interactions	386
	H.1 A Very Short Introduction to Nonstandard Analysis	386
	H.2 Point Interactions Using Nonstandard Analysis	391
I	Elements of Probability Theory	396
J	Relativistic Point Interactions in One Dimension	399
	References	413
	Index	441