## **Contents**

	_	ort Rays and Applications to Hamilton-Jacobi			
		ons	1		
St	efano	Bianchini and Matteo Gloyer			
1	Introduction				
2	Settings				
3	Disintegration				
4	Hamilton-Jacobi Equation and Monotonicity				
5		larity Properties of L-Conjugate Functions			
	_	Optimal Rays	8		
	5.1	Rectifiability Property of Jumps	9		
	5.2	Some Approximations	10		
	5.3	Fundamental Example	10		
	5.4	Divergence Estimate	10		
6	Jacobian Estimates		11		
	6.1	Disintegration of the Lebesgue Measure	12		
	6.2	Regularity of the Jacobian and Applications	13		
	6.3	Divergence Formulation	14		
Re	eferenc	es	15		
Fu	ınctio	nal Inequalities and Dynamics	17		
	A. Ca				
1		duction	17		
-	1.1	The Sharp Hardy-Littlewood-Sobolev Inequality	19		
	1.2	Duality and Functional Inequalities	21		
	1.3	A Limiting Case of the HLS Inequality	25		
	1.4	The Two Dimensional Energy Integral	26		
	1.5	The Logarithmic HLS Inequality for $d = 2$	30		
	1.6	The Dual of the Sharp HLS Inequality for $\mathbb{R}^2$	32		
	1.7	The Legendre Transforms of $S_M$ and $J_M$	34		
	1.1	THE LEGERAL TRANSPORMS OF DM and JM	.54		

digitalisiert durch NATION BIBLIO

x Contents

2		e Evolution Equations for Which These Sharp	
	Func	etional Inequalities Are Relevant	40
	2.1	The Keller-Segel Model	40
	2.2	Consequence of the Logarithmic HLS Inequality:	
		No Blow-Up for $M < 8\pi$	42
	2.3	Evolution of the Second Moment: Blow-Up for $M > 8\pi$	43
	2.4	Equilibrium Solutions for $M = 8\pi$	44
	2.5	The Critical Fast Diffusion Equation	
		as Gradient Flow of an Entropy	44
	2.6	Monotonicity of the Logarithmic HLS	
		Functional Under Critical Fast Diffusion	47
3	Com	peting Symmetries: A Dynamical Proof of the HLS	
		uality and More	51
	3.1	Rearrangement	53
	3.2	The Riesz Rearrangement Inequality	55
	3.3	The Main Ideas of Competing Symmetries	
	0.0	as Applied to the HLS Inequality	56
	3.4	A General Competing Symmetries Theorem	58
	3.5	Application to the HLS Inequality	61
	3.6	Cases of Equality	62
	3.7	Other Applications of Competing Symmetries	63
4		eralized Brascamp-Lieb Inequalities	64
-	4.1	The Brascamp-Lieb Inequality with a Gaussian	O I
		Reference Measure	66
	4.2	A Brascamp-Lieb Inequality the Sphere	67
	4.3	A Brascamp-Lieb Inequality the Permutation Group	67
	4.4	Brascamp-Lieb Inequalities and Entropy	69
	4.5	Proof of the BL Inequality on the Sphere	73
5		Slow Diffusion Equation	77
J	5.1	About the Thin Film Equation	77
	$5.1 \\ 5.2$	Equipartition of Energy	81
	5.3	For Which Class of Solutions?	83
D.		ror which class of Solutions:	84
100	ner en (	.cs	04
$\mathbf{D}_{\mathbf{i}}$	iffere	ntial, Energetic, and Metric Formulations	
fo	r Rat	e-Independent Processes	87
Al		er Mielke	
1	Intro	duction	87
2	Basic	es of Rate-Independent Systems	91
	2.1	Definition of Rate Independence	91
	2.2	Differentiable Formulations and the Decomposition	
		into Elastic and Dissipative Parts	92
	2.3	Some Canonical Examples	93
	2.4	The Basic a Priori Estimates	95
	2.5	Energetic Formulation of Generalized Gradient Flows	97

Contents xi

	2.6	Solution Concepts in the One-Dimensional Case	100
	2.7	Infinite-Dimensional Examples	105
3	Energetic Solutions		
	3.1	Abstract Setup of the Problem	107
	3.2	The Time-Incremental Minimization Problem	109
	3.3	Statement of the Main Existence Result	112
	3.4	Jump Conditions for Energetic Solutions	114
	3.5	On Compatibility Conditions (C1) and (C2)	116
	3.6	Proof of Theorem 3.4	117
	3.7	$\Gamma$ -Convergence for Sequences of Rate-Independent	
		Systems	124
4	Rate-	Independent Systems in Banach Spaces	128
	4.1	The Basic Banach-Space Setup	128
	4.2	Differential, CD, and Local Solutions	132
	4.3	Systems with Convexity Properties	136
	4.4	Parametrized Solutions Via the Vanishing-Viscosity	
		Approach	139
	4.5	BV Solutions and Optimal Jump Paths	150
	4.6	Weak BV Solutions and Time-Varying	
	1.0	Dissipation Distances	158
5	Metri	ic Formulations	160
Ū	5.1	Metric Velocity, Slope, and Evolution	160
	5.2	Parametrized Metric Solutions	162
	5.3	Metric BV Solutions	166
Re		es	167
		l Transport and Curvature	171
		Figalli and Cédric Villani	
1		duction	171
2		of Metric Geometry	172
	2.1	Length	172
	2.2	Length Spaces	172
	2.3	Geodesics	173
	2.4	Riemannian Manifolds	174
	2.5	Riemannian Distance and Volume	175
	2.6	Differential and Gradients	175
	2.7	Geodesics in Riemannian Geometry	176
	2.8	Exponential Map and Cut Locus	176
	2.9	First Variation Formula and (Super)Differentiability	
		of Squared Distance	177
	2.10	Hessian and Second Order Calculus	179
	2.11	Variations of Geodesics and Jacobi Fields	179
	2.12	Sectional and Ricci Curvatures	180
	2.13	Interpretation of Ricci Curvature Bounds	181
	2.14	Why Look for Curvature Bounds?	182

xii Contents

	2.15	Stability Issue and (Measured) Gromov-Hausdorff	
		Convergence	183
3	Solut	ion of the Monge Problem in Riemannian Geometry	185
	3.1	The Monge Problem with Quadratic Cost	185
	3.2	Existence and Uniqueness	
		on Compact Manifolds	185
	3.3	c-Convexity and c-Subdifferential	186
	3.4	Sketch of the Proof Theorem 3.2	187
	3.5	Interpretation of the Function $\bar{\psi}$	190
4	Syntl	netic Formulation of Ricci Bounds	191
	4.1	The 2-Wasserstein Space	191
	4.2	Geodesics in $P_2(M)$	191
	4.3	Approximate Geodesics in Wasserstein Space	192
	4.4	Reformulation of Ric $\geq 0$	193
	4.5	Application: Stability	194
5	The S	Smoothness Issue	195
	5.1	The PDE	196
	5.2	Obstruction I: Local Geometry	196
	5.3	Obstruction II: Topology of the c-Subdifferential	197
	5.4	Conditions for the Connectedness of $\partial^c \psi$	198
	5.5	The Ma-Trudinger-Wang Tensor	201
	5.6	Invariance of S	204
	5.7	Relation to Curvature	204
	5.8	The Ma-Trudinger-Wang Condition	205
	5.9	Local to Global	206
	5.10	A Smoothness Result	209
6		p and Perspectives	211
•	6.1	The Curvature-Dimension Condition	211
	6.2	Open Problem: Locality	212
	6.3	Ricci and Diffusion Equations	212
	6.4	Discretization	213
	6.5	Smoothness	213
7	Selected References		214
	7.1	Links Between Optimal Transport and Ricci Curvature	214
	7.2	Optimal Transport and Ricci Flow	215
	7.3	Discrete Ricci Curvature	215
	7.4	Smoothness of Optimal Transport,	
	•••	Cut Locus and MTW Tensor	215
Re	ferenc	es	215
	.010110		<b>_</b> 10
T :	et of	participants	219
*11;	o Ot	par incipating	Z13