

AMERICAN UNIVERSITY STUDIES

The Concept of Logical Consequence

An Introduction to Philosophical Logic

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Chapter 1

Introduction

This book is an inquiry into the concept of logical consequence, arguably the central concept of logic. We take logical consequence to be a relation between a given set of sentences and the sentences that logically follow. One sentence is said to be a logical consequence of a set of sentences, if and only if, in virtue of logic alone, it is impossible for the sentences in the set to be all true without the other sentence being true as well. The central question to be investigated here is: what conditions must be met in order for a sentence to be a logical consequence of others?

One historically significant answer derives from the work of Alfred Tarski, one of the greatest logicians of the twentieth century. In Chapter 2, we distinguish features of the ordinary, informal concept of logical consequence using some of Tarski's work, particularly his seminal (1936) paper on logical consequence. Here Tarski uses his observations of the salient features of what he calls the common concept of logical consequence to guide his theoretical development of it. We shall develop Tarski's observations of the criteria by which we intuitively judge what follows from what, and which Tarski thinks must be reflected in any theory of logical consequence.

After presenting his theoretical definition of logical consequence, which is the forerunner of the modern, model-theoretic definition, Tarski asserts in his (1936) paper that it reflects the salient features of the common concept of logical consequence. This assertion is not obvious, and Tarski defends it nowhere in his published writings. This raises the particular issues of whether Tarski's informal characterization of the common concept of logical consequence is correct, and whether it is reflected in his theoretical definition. The more general issues raised are: how do we justify a theoretical definition of logical consequence? What role should the informal concept play?

We shall answer these questions with respect to the model-theoretic and the deductive-theoretic characterizations of logical consequence for first-order languages. They represent two major theoretical approaches to making the common concept of logical consequence more precise. Chapter 2 shall motivate both approaches by considering them as natural developments of the ordinary, informal characterization. This shall set the context for our critical evaluation of these two approaches to characterizing logical conse-

quence. After introducing some set-theoretic concepts used in the book and a simple first-order (extensional) language M in Chapter 3, (classical) logical consequence shall be defined for M model-theoretically in Chapter 4, and deductive-theoretically (a natural deduction system N is given) in Chapter 5. I account for their status as definitions, and sketch how they work in determining what follows from what. Also, there are accounts of what models and deductive apparatuses are, and, what, exactly, they represent when used to fix the logical consequence relation.

Both Chapters 4 and 5 consider methodological criticism of the model-theoretic and deductive-theoretic approaches. In particular, we consider the adequacy of models and deductive apparatuses as tools for defining logical consequence, and these considerations are used to answer the two questions posed above: how do we justify a theoretical definition of logical consequence? What role should the informal concept play? Also, in Chapters 4 and 5, there is some criticism of classical logic. Both types of criticism (methodological and logical) not only motivate consideration of alternative logics, but also suggest revisions to the Tarskian understanding of the informal concept of logical consequence introduced in Chapter 2.

While most logicians accept the model-theoretic and deductive-theoretic characterizations of logical consequence for extensional languages, there is less agreement on the pre-theoretic notion these technical definitions are supposed to represent, and little discussion about whether they actually do represent it adequately. Almost all of the formal logic textbooks written for the book's intended audience give an ordinary, informal characterization of logical consequence either in the introduction or at the beginning of the first chapter. Unfortunately, the informal characterization of logical consequence typically amounts to a mere sketch which is either insufficient for clarifying the status of the technical characterizations that follow or conflicts with them. The book's focus on the concept of logical consequence, its introductory manner of presentation, and its monograph-length, make it ideal for the intended audience as a means for clarifying the status and aims of the technical characterizations of logical consequence, and for highlighting their relationship to the informal concept of logical consequence which motivates them. This enhances understanding of not only the status of the model-theoretic and deductive-theoretic characterizations of logical consequence, but also deepens our understanding of criteria for evaluating them.

The book's intended audience matches the audiences of other introductions to philosophical logic such as Haack (1978), Sainsbury (1991), and

Read (1995). Like these classics, this book is written at a level that makes it beneficial to advanced undergraduates with exposure to introductory formal logic, graduate students, and professional philosophers planning to self-educate themselves about the philosophy of logical consequence and for whom this book is only a first step. What distinguishes this book is its approach to thinking about logical consequence. It is tightly organized around the informal concept of logical consequence, and its relationship to the more technical model-theoretic and deductive-theoretic characterizations. I am unaware of any introduction to philosophical logic devoted to motivating the technical characterizations of logical consequence by appealing to the informal concept of logical consequence, and evaluating the former in terms of how successfully they capture the central features of the latter. As with the above three books when first published, the freshness of this book's approach to studying logical consequence and its engagement with themes in the recent literature should make it of interest to specialists working in the philosophy of logic.

The goal of realizing the envisioned length of the book has, of course, had an expository impact. In order to forgo lengthy exegetical analysis, ideas and arguments from the literature are typically presented in a distilled form. Also, since references and discussion are confined to the main text, there are no footnotes. More importantly, I have been careful in choosing where to be argumentative (as in my defense of the Tarskian model-theoretic characterization of logical consequence against criticism) and where to remain agnostic (as with respect to the issue of the nature of a logical constant and whether the meanings of logical constants should be identified with their truth-conditional properties or their inferential properties). I've chosen to be argumentative in those places where I believe that I have the space to be persuasive. In many places where the discussion is more expository and less argumentative, I have developed topics to the point that satisfies the stated goals of the relevant section. I certainly realize that on pretty much every topic covered in the book, much more can be usefully said. I have at points in the text provided the reader with references that extend the book's discussion in various ways.

There are references to Tarski's work throughout the book's discussion of logical consequence. While I do not believe that my reading of Tarski is controversial, the desire not to lengthen the book prohibits defense of my interpretation of Tarski. To be clear, this is not a book about Tarski. Rather, some of Tarski's writings are used as a platform for the book's discussion of

logical consequence. The thoughts of other logicians such as Dummett, Gentzen, and Frege, are also used towards this end. The trajectory of the discussion is squarely aimed at the model-theoretic and deductive-theoretic approaches to logical consequence, and their relationship to the informal concept of logical consequence. Even though the focus of the book is on logical consequence, it is studied in a way that allows it to serve as an introduction to philosophical logic. Its emphasis on the informal concept of logical consequence and its relationship to the more technical model-theoretic and deductive-theoretic approaches highlights and sharpens in a unique way other issues central in the philosophy of logic such as the nature of logic, logical constants, and logical necessity. Introducing issues in the philosophy of logic from the perspective of a study of logical consequence will illustrate how these issues are related, and why they are significant for understanding logical consequence.