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Logos and Máthēma

Studies in the Philosophy of Mathematics
and History of Logic

Foreword

The volume contains 20 essays devoted to the philosophy of mathematics and the history of logic. They have been divided into four parts. Part 1 contains papers considering general philosophical problems of mathematics. In the essay "Mathematical Knowledge" basic epistemological problems of mathematics are considered. Main doctrines in the epistemology of mathematics have been presented and analyzed. The interrelations between logic and philosophy of mathematics as well as some current tendencies in the philosophy of mathematics have been studied. The essay "On the Power and Weaknesses of the Axiomatic Method" discusses the meaning of the axiomatic method for the methodology of mathematics. In "Remarks on the Mathematical Universe" the problem of the existence and the nature of mathematical entities is considered. Various conceptions that appeared in the history are presented and examples of mathematicians and logicians declaring for them are indicated. Consequences of those conceptions for doing mathematics are considered. In the last essay in this part "Structuralism and Category Theory in the Contemporary Philosophy of Mathematics" set-theoretical (Bourbaki-style) and category-theoretical approaches to structuralism in the philosophy of mathematics are compared. Advantages and disadvantages of them are indicated.

Part 2 is devoted to problems concerning Hilbert's program and the influence on it of the discovery of the incompleteness phenomenon. In the essay "Hilbert's Program: Incompleteness Theorems vs. Partial Realizations" the question whether Gödels' incompleteness theorems did reject Hilbert's program is studied. Generalizations and strengthenings of Gödel's results as well as generalized and relativized Hilbert's programs and their meaning for the philosophy of mathematics are considered. The essay "On the Distinction Proof–Truth in Mathematics" contains some historical, philosophical and logical considerations connected with the distinction between proof and truth in mathematics. The crucial rôle of Gödel's incompleteness theorems as well as of the undefinability of truth vs. definability of provability and the rôle of finitary vs. infinitary methods are stressed. The problem of the necessity of extending the available methods by new rules of inference and new axioms is also considered. The discovery of the incompleteness phenomenon destroyed the old conviction that axiomatic method is the ideal method for mathematics. Therefore it was not immediately accepted by logicians. Reactions to this discovery are considered in the next paper. The essay "Gödel's Incompleteness Theorems and Computer Science" indicates some applications of Gödel's results to the discussion of problems of computer science. In particular the problem of relations between the mind and machine (arguments by J. J. C. Smart and J. R. Lucas), Gödel's opinion on this issue and

some interpretations of the incompleteness theorems from the point of view of the information theory are presented. Though it seems that human mind is not fully equivalent to a machine (computer), nevertheless some of its functions can be mechanized. Next essay "The Present State of Mechanized Deduction, and the Present Knowledge of Its Limitations" tells about attempts to develop procedures of mechanized deduction. It indicates also various limitations of them.

One of the aims of Hilbert's program was to show that the classical mathematics (referring to the actual infinity) is safe and free of any inconsistencies. Hence the attempts to prove the consistency of basic mathematical theories by finitistic methods undertaken in Hilbert's school (Ackermann, von Neumann). Gödel's results show that new non-finitistic methods must be applied here. Those problems are considered in the paper "On Proofs of the Consistency of Arithmetic". The paper "Decidability vs. Undecidability. Logico-Philosophico-Historical Remarks" presents the decidability problem from a philosophical and historical perspective. It indicates also basic mathematical and logical results concerning (un)decidability of particular theories and problems. In the paper "Undefinability of Truth. The Problem of Priority: Tarski vs. Gödel" it is argued that Tarski obtained the theorem on the undefinability independently from Gödel though he made clear his indebtedness to Gödel's methods. On the other hand Gödel was aware of the formal undefinability of truth in 1931 but he did not publish this result – reasons for that are considered. The problem of definability and undefinability of the concepts of satisfaction and truth from a more technical point of view is considered in the essay "Troubles With (the Concept of) Truth in Mathematics" closing Part 2.

Parts 3 and 4 are devoted to the work of Polish logicians and mathematicians in the philosophy of mathematics and in logic. Part 3 begins by an essay presenting the philosophical system of deep and interesting but unfortunately rather forgotten and underestimated Polish philosopher and mathematician Józef Maria Hoene-Wroński. In the essays "Philosophical Reflection on Mathematics in Poland in the Interwar Period" and "Philosophy of Mathematics in the Warsaw Mathematical School" the views and tendencies of the most outstanding representatives of Lvov-Warsaw Philosophical School and of the founders of Polish Mathematical School are presented and analyzed. The problem whether those views had any influence on logical and mathematical research is considered. Philosophical views concerning mathematics of Andrzej Mostowski, an outstanding representative of the second generation of the Lvov-Warsaw School, are presented and discussed in the next essay.

The last Part 4 contains three essays devoted to Polish mathematical logic. The first one "Stanisław Piątkiewicz and the Beginnings of Mathematical Logic in Poland" presents information on the life and work of Stanisław Piątkiewicz. His *Algebra w logice* (1888) contained an exposition of the algebra of logic and its use in representing syllogisms. This was the first original Polish publication

on symbolic logic (it appeared 20 years before analogues works by Łukasiewicz and Stamm). In the next essay contribution of Polish logicians to the recursion theory is presented. The final essay is devoted to logical investigations at the University of Poznań in the period 1945–1955. It is considered whether there was any continuation of the Lvov-Warsaw Logical School in Poznań.

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