1 Introduction

The job of science is to enable the inquiring mind to feel at home in a mysterious universe.

Lewis Carroll Epstein, *Relativity Visualized*

1.1 The Meaning of Relativity

The theory of relativity (special and general) is one of the cornerstones of modern physics. Its basic element is the principle of relativity. The word "relativity" here reflects only one, although very important, aspect of this principle: certain physical characteristics of a system are relative, in the sense that a numerical value of such characteristic measured by one observer may be different from the value measured by another observer moving with respect to the first one. The second aspect, inseparable from the first one, is that all laws of Nature are independent of the observer's motion. This statement reflects the "absolute" aspect of the principle of relativity, namely, that the *physical laws are the same for all observers*. And the two aspects are inseparable because one directly follows from the other. Indeed, the relativity of motion ("the states of rest and motion do not have the absolute meaning") follows immediately from absoluteness of natural laws (they are the same regardless of the state of motion of an observer).

We will start here with the relativity aspect. And a good starting point may be the discussion of such familiar characteristics of motion as velocity. Even a person with only rudimentary education can easily understand that velocity is a relative characteristic. If you are riding on a train and see another passenger passing from the rear of the train car to her seat in the front, you could estimate her velocity as about 2 miles/h. But an observer outside the train may estimate her velocity as 42 miles/h, owing to the additional 40 miles/h made by the train.[1] The velocity of an object acquires exact meaning only when we specify relative to what it is measured. In this respect, it is a "flexible" characteristic. An object that is perceived by a ground-based observer to be moving is at rest to another observer moving together with this object. A third observer, moving in the same direction, but faster than the second one, will see the

1) We will see later that this simple addition of velocities is only an approximation to the more general rule.

Special Relativity and How it Works. Moses Fayngold

ISBN: 978-3-527-40607-4

same object moving in the opposite direction. We will call such quantities as velocity "observer-dependent," or relative.

Not all physical quantities are relative, however. Some of them are observer-independent, or absolute. Here is a simple example: if a car with three passengers has a velocity 45 miles/h, then the fact of it having this velocity is of a quite different category than the fact of it having three passengers inside. The latter is absolute because it is true for anyone regardless of one's state of motion. The former is relative because it is only true for those standing on the ground. But it is false, say, for a driver in another car moving along the same straight road. The driver will agree with you on the number of passengers in the first car but disagree on its velocity. He may hold that the first car has zero velocity because it has always been at the same distance from him.

Who is right – you or the second driver? Both are. And there is no contradiction here, because each observer relates what he sees to his own "reference frame."

Moreover, even one and the same observer can measure different velocities of the same object, depending on the observer's state of motion. A policeman in a car, using radar for measuring speeds of moving objects, will record two different values for the velocity of a vehicle, if he measures this velocity first time when his own car just stands on the road and the second time when his car is moving. We emphasize that nothing happens to the observed vehicle, it remains in the same state of motion with constant speed on a straight highway. And yet the value of this speed as registered by the radar is different for the two cases.

We thus see that the value of a speed does not by itself tell us anything. It only becomes meaningful if you specify *relative to what* this speed is measured. This is what we mean by saying that speed (more general, velocity) is a relative physical quantity.

Understanding the relative nature of some physical quantities (and absolute nature of some others) is the first step to acquiring the main ideas of special relativity.

Let us start with the widespread public perception of the theory of relativity: "Einstein has proved that everything is relative. Even time is relative."

One of these statements is true and profoundly deep; the other one is totally misleading.

The true statement is: *time is relative*. The realization of relative nature of time was a revolutionary breakthrough in our understanding of the world.

The wrong statement in the above "popular" account of relativity is that *everything* is relative. We already know that, for instance, the number of passengers in a car (or the chemical composition of a certain material) is not relative. One of the most important principles in relativity is that, together with natural laws, *certain physical quantities are absolute (invariant)*. One of such invariable quantities is the speed of light in vacuum. Also, a certain combination of time and distance turns out to be invariant. We will discuss these absolute characteristics in the next chapters. They are so important that we might as well call the theory of relativity the theory of absoluteness. It all depends on which aspect of the theory we want to emphasize.

We will now discuss in more details the relativity aspect, but keep in mind that, as emphasized above, its essence is the absolute status of the laws of Nature.

Let us first recall the classical principle of relativity in mechanics. Suppose you are inside a train car that moves uniformly along a straight track. If the motion is smooth enough, then, unless you look out of the window, you cannot tell whether the train moves or is at rest on the track. For instance, if you drop a book, it will fall straight down with acceleration, as it would do on the stationary platform. It will hit the floor near your feet, as it would do on the platform. If you play billiards, the balls will move, and collide, and bounce off in precisely the same manner as they do on the platform. And all other experiments will be indistinguishable from those on the platform. There is no way to tell, whether you are moving or not, by performing mechanical tests. This means that the states of rest and uniform motion are equivalent for mechanical phenomena. There is no intrinsic, fundamental difference between them. This general statement was formulated by Galileo and it came to be known as his principle of relativity. According to this principle, the statement “My train is moving” has no absolute meaning. Of course, you can find out that it is moving, the moment you look out of the window. But the moment you do it, you start referring all your observations to the platform. You then can say: “My car is moving relative to the platform.” Platform constitutes your reference frame in this case. But you may as well refer all your data to the car you are in. Then the car itself will be your reference frame, and you may say: “My car is at rest, while the platform is moving relative to it.” Now, pit the last two quoted statements against each other. They seem to be in contradiction, but they are not, because they refer to different reference frames. Each statement is meaningful and correct, once you specify the corresponding frame of reference.

We see that the concept of reference frame plays a very important role in our description of natural phenomena. We can even reformulate the principle of relativity in terms of reference frames. To broaden the pool of examples (and make the further discussion more rigorous!), we will now switch from jittering trains, and from spinning Earth with its gravity, far into deep space. A better, and more modern, realization of a suitable reference frame would be a nonrotating spaceship with its engines off, coasting far away from Earth or other lumps of matter. Suppose that initially the ship just hangs in space, motionless with respect to distant stars. You may find this an ideal place to check the basic laws of mechanics. You perform corresponding experiments and find all of them confirmed to even higher precision than those on Earth.

If you release a book, it will not go down; there is no such thing as “up” or “down” in your spaceship, because there is no gravity in it. The book will just hang in air close by you. If you give it an instantaneous push, it will start moving in the direction of the push. Inasmuch as you can neglect air resistance, the book will keep on moving in a straight line with constant speed, until it collides with another object. This is a manifestation of Newton’s first law of motion – the famous law of inertia. Then you experiment with different objects, applying to them various forces or combination of forces. You measure the forces, the objects’ masses, and their response to the forces. In all cases, the results invariably confirm Newton’s second law – the net force accelerates an object in the direction of the force, and the magnitude of the acceleration is such that its product by the mass of the object equals the force. This explains why the released book does not go down – in the absence of gravity it does not know where “down” is. With no gravity, and possible other forces balanced, the net force on the book and thereby its acceleration is zero. Then you push against the wall of your compartment and immediately find yourself being pushed back by the wall and flying away from it. This is a manifestation of Newton's third law: forces always come in pairs; to every action there is always equal and opposite reaction.

Let us now stop for a while and make a proper definition. Call a system where the law of inertia holds, an inertial system or inertial reference frame. Then you can say that your ship represents an inertial system. So does the background of distant stars relative to which the ship is resting.

Suppose now that you fall asleep and during your sleep the engines are turned on. The spaceship is propelled up to a certain velocity, after which the engines are turned off. You are still asleep, but the ship is now in a totally different state of motion. It has acquired a velocity relative to the background of stars, and it keeps on coasting with this velocity due to inertia. The magnitude of this velocity may be arbitrary. But even if it is nearly as large as that of light, it will not by itself affect in any way the course of events in the ship. After you have woken up and checked if everything is functioning properly, you do not find anything unusual. All your tests give the same results as before. The law of inertia and other laws hold as they had done before. Your ship therefore represents an inertial reference frame as it had been before. Unless you look outside and watch the "sky" or measure the spectra of different stars, you would not know that your ship is now in a *different state of motion* than it had originally been. The reference frame associated with the ship is therefore also different from the previous one. But, according to our definition, it remains inertial.

What conclusions can we draw from this? First, any system moving uniformly relative to an inertial reference frame is also an inertial reference frame. Second, all the inertial reference frames are equivalent with respect to all laws of mechanics. The laws are the same in all of them. The last statement is the classical (Galilean) principle of relativity expressed in terms of the inertial reference frames.

The classical principle of relativity is very deep. It seems to run against our intuition. In this era of computers and space exploration, I have come across a few students in my undergraduate physics class who argued that if a passenger in a uniformly moving subway car dropped an apple, the apple would not fall straight down, but rather would go somewhat backwards. They reasoned that while the apple is falling down, the car is being pulled forward from under it, which causes the apple to hit the floor closer to the rear of the car. This argument, which overtly invokes the platform as a fundamental reference frame, overlooks one crucial detail: before being dropped, the apple in the passenger's hand was moving forward together with the car. This preexisting component of the motion persists in the falling apple due to inertia and exactly cancels the effect described by the student, so that the apple as seen by an observer in the car will go down strictly along its vertical path (Figure 1.1). This conclusion is confirmed by innumerable observations of falling objects in moving cars. It is a remarkable psychological phenomenon that sometimes not even such strong evidence as direct observation can overrule the influence of a more ancient tradition of thought. About a century and a half ago, when the first railways and trains appeared, some people were afraid to ride in them because of their great speed. The same story repeated at the emergence of aviation. Many people were afraid to board a plane not only because of the altitude of flight, but also because of its

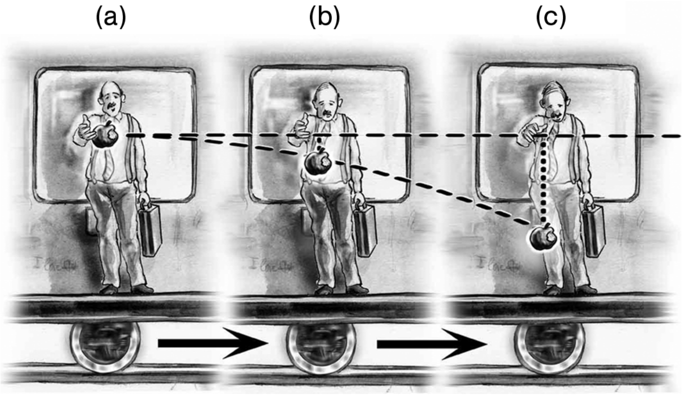

Figure 1.1 The fall of an apple in a moving car as observed from the platform: (a), (b), and (c) are the three consecutive snapshots of the process. The passenger sees the apple fall vertically, while it traces out a parabola relative to the platform. The shape of trajectory turns out to be a relative property of motion.

great speed. Apart from the fear of a *collision* at high speed, it might have been the fear of the speed itself. Many believed that something terrible would happen to them at such a speed. It took a great deal of time and new experience to realize that speed itself, no matter how great, does not cause any disturbance in the regular patterns of natural events so far as velocity remains constant. It is the *change* of velocity (acceleration) during braking, collision, or turn that can be felt and manifests itself inside of a moving system. If you are in a car that is slowing down, you can immediately tell this by the appearance of a force that pushes you forward. Likewise, if the car accelerates, everything inside experiences a force in the backward direction. It is precisely because of these forces that I wanted you to fall asleep during the acceleration of the spaceship. Otherwise, you would have immediately noticed the appearance of a new force and known that your ship was changing its state of motion, which I did not want you to do.

A remarkable thing about this new force is that it does not fit into the classical definition of a real force. It appears to be real because you can observe and measure it; you have to apply a real force to balance it; when unbalanced, it causes acceleration, as does any real force; it is equal to the product of a body's mass and acceleration, as is any real unbalanced force. In this respect, it obeys Newton's second law. Yet, it appears to be fictitious if you ask a question: Who (or what) exerts this force? Where does it come from? Then you realize that it, unlike all other forces in Nature, does not have a physical source. It does not obey Newton's third law, because *it is not a part* of an action–reaction pair. You cannot find and single out a material object producing this force, not even if you search out the whole universe. Unless, of course, you prefer to consider the whole universe becoming its source when the universe is accelerated past your frame of reference.

The new force has been called the inertial force – for a good reason. First, it is always proportional to the mass of a body it is applied to – and mass is the measure of

Figure 1.2 A chandelier in an accelerated car. To Alice, the tension force in the deflected chain acquires a horizontal component causing the chandelier to accelerate at the same rate as the car. Tom explains the deflection of the chandelier as the result of the inertial force. This force balances the horizontal component of the chain's tension.

the body's inertia. In this respect, it is similar to the force of gravity. Second, its origin can be easily traced to a manifestation of inertia. Imagine two students, Alice and Tom. They both observe the same phenomenon from two different reference frames. Tom is inside a car of a train that has just started to accelerate, while Alice is on the platform. Alice's reference frame is, to a very good approximation, inertial, while Tom's one is not. Tom looks at a chandelier suspended from the car's ceiling. He notices that the chandelier deflects backward during acceleration. He attributes it to a fictitious force associated with the accelerating universe. Alice sees the chandelier from the platform through the car's window (Figure 1.2), but she interprets what she sees quite differently. "Well," she says, "this is just what should be expected from the Newton's laws of motion. The unbalanced forces are exerted on the car by the rails and, may be, by the adjacent cars, causing the car to accelerate. However, the chandelier, which hangs from a chain, does not immediately experience these new forces. Therefore, it retains its original state of motion, according to the law of inertia, which holds in my reference frame. At the start, the chandelier accelerates back relative to the car only because the car accelerates forward relative to the platform. This transitional process lasts until the deflected chain exerts sufficient horizontal force on the chandelier." "Finally, Alice concludes, this force will accelerate the chandelier relative to the platform at the rate of the car, and there will be no relative acceleration between the car and chandelier." All the forces are accounted for in Alice's reference frame. In Tom's frame of reference, the force of inertia that keeps the chandelier with the chain off the vertical is felt everywhere throughout the car but cannot be accounted for. This state of affairs tells Tom that his car is accelerating.

Figure 1.3 Water in an accelerated fish tank. The rear wall of the tank rushes upon the water, raising its adjacent surface, while the front wall accelerates away from the water, giving it an extra room in front, which causes the water there to sink. To Tom, tilt of the water surface is caused by the inertial force. The tilted chain of the chandelier makes the right angle with the tilted water surface.

Tom has also taken the pain of bringing along an aquarium with fish in it. When the train starts accelerating, both Tom and Alice see the water in the aquarium bulge at the rear edge and subside at the front edge, so that its surface forms an incline (Figure 1.3). Alice interprets it by noticing that the rear wall of the aquarium drives the adjacent layers of water against the front layers, which tend to retain their initial velocity. This causes the rear layers to rise; in contrast, the front layers sink because the front wall of the fish tank accelerates away from them. Thus, the water surface tilts.

Tom does not see any accelerated motions within his car, but he feels the horizontal force pushing him toward the rear. "Aha," Tom says, "this force seems to be everywhere indeed. It pushes me and the chandelier back, and now I see it doing the same to water. It is quite similar to the gravity force, but it is horizontal and seems to have no source. Its combination with the Earth-caused gravity gives the net force tilted with respect to the vertical line." Being as good a student as Alice, Tom knows that the water surface always tends to adjust itself so as to be perpendicular to the net force acting on it. Since the latter is now tilted toward the vertical, the water surface in the aquarium becomes tilted to the horizontal by the same angle. The only trouble is that there is no physical body responsible for the horizontal component of the net force. "This indicates," Tom concludes, "that horizontal component is a fictitious inertial force caused by acceleration of my car."

In a similar way, one can detect a rotational motion, because the parts of a rotating body accelerate toward its center. We call it centripetal acceleration. For instance, we could tell that the Earth is rotating even if the sky were always cloudy so that we would be unable to see the Sun, Moon, or stars. That is, we could not "look out of the window." But we do not have to. Many mechanical phenomena on Earth betray its rotation. The Earth is slightly bulged along the equator and flattened at the poles. A free-falling body does not precisely fall along the vertical line (unless you experiment at one of the geographical poles). A pendulum does not swing all the time in one plane. Many rivers tend to turn their flow. Thus, in the Northern Hemisphere, rivers are more likely to have their right banks steep and precipitous and the left ones shallow. In the one-way railways, the right rails are being worn out faster than the left ones because the rims of the trains' wheels are pressed mostly against the right rail. In the Southern Hemisphere, the situation is the opposite. It is easier to launch a satellite in the east direction than in the north or south, let alone west direction. All these phenomena are manifestations of the inertial forces.

We will illustrate the origin of these forces for a simplified model of a train moving radially on a rotating disk. Suppose that the train is moving down a radial track toward the center of the disk and you observe this motion from an inertial stationary platform (Figure 1.4). At any moment the instantaneous velocity of the train relative to the platform has two components: radial toward the center and transverse, which is due to the local rotational velocity of the disk. The peripheral parts of the disk have larger rotational velocity than the central parts. As the train moves toward the center, it tends, following the law of inertia holding on the platform, to retain the larger

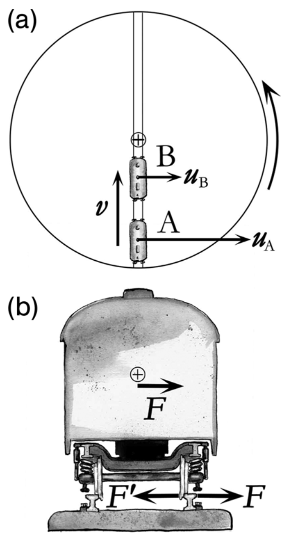

Figure 1.4 Schematic representation of the inertial forces acting on a moving car in a rotating reference frame. (a) View from above. The train moves from A to B with the speed v. Because of inertia, the train tends to transport its original rotational velocity u_A from A to B. Since u_A is greater than u_B, the train experiences transverse inertial force F. (b) View from behind. The force F is balanced by force F'.

rotational velocity "inherited" from the peripheral parts of the disk. This would immediately cause the derailment onto the right side of the track, had it not been for the wheels' rims that hold the train on the rails. The same effect causes the overall asymmetry between the left and the right banks of the rivers. We thus see that these phenomena are, in fact, manifestations of the inertia. Their common feature is that they permeate all the space throughout an accelerated system and cannot be attributed to an action of a specific physical body. Because of them, the Earth can be considered as an inertial system only to a certain approximation. Careful observation reveals the Earth's rotation without anyone having ever to look up in the sky.

All these examples show that inertial systems in classical physics form a very special class of moving systems. The world when looked upon from such a system looks simpler because there are no inertial forces. You can consider any inertial system as stationary by choosing it to be your reference frame without bringing along any inertial forces. There is no intrinsic physical difference between the states of rest and uniform motion. All other types of motion are absolute in a sense that Nature provides us with the criterion that distinguishes one such motion from all the others. We can also relate all observational data to an accelerated system and consider it motionless. But there are intrinsic physical phenomena (inertial forces) that reveal its motion relative to an inertial reference frame. Not only can we detect this motion without "looking out of the window," but we can also determine precisely all its characteristics, including the magnitude and direction of acceleration, the rate of rotation, and the direction of the rotational axis.

We thus arrive at the conclusion that Nature distinguishes between inertial and accelerated motions. It does not at all mean that the theory cannot describe accelerated motions. It can, and we will see the examples of such description further in the book. The special theory of relativity can even be formulated in arbitrary accelerated and therefore noninertial reference frames [19]. But the description of motion in such systems is far less straightforward, to a large extent because of the appearance of the inertial forces. The general theory of relativity reveals deep connections between the inertial forces in an accelerated system and gravitation. We will in this book be concerned mostly with special relativity and make a very brief outline of basic ideas of general relativity in the last chapter.

Discussion

Here we want to mention another misconception of relativity of time among the general public. It can be expressed as an extension of a passerby's remark quoted above: "Einstein has proved that time is relative. One minute or one million years makes no difference. So relax and take it easy."

While the first statement here is true, the second one is a good example of what can happen to an idea when one takes it to extremes and on the way changes its meaning. In the given case, the result is what philosophers of science call "relativistic nihilism." The corresponding view is not even wrong. It is utterly meaningless. The relativity of time refers to the duration of a process observed or measured from *different* reference frames. By contrast, the last statement in the quotation is about duration of a process in *the same* reference frame. And in this case saying that there is no difference between one minute and one million years is equivalent to saying that there is no difference between one cent or one million dollars in your bank account. If this passerby is rich, but someday (God forbid!) loses his millions, with only one cent left on him, let him then use his own good advice – relax and take it easy.

Another important point relevant to the above discussion is the difference between a reference frame and a coordinate system. A reference frame is a physical object (usually – but not always – a sufficiently rigid body) to which we refer our measurements and observations. It may be a car, a plane, a spaceship, a planet (say, our Earth), a galaxy, or even a set of galaxies.

A coordinate system is a way we specify a position by assigning to it a set of numbers (coordinates). We can use an infinite variety of *different* coordinate systems associated with the *same* reference frame. The most familiar are the Cartesian, spherical, and cylindrical coordinate systems. The most commonly used is a Cartesian system – a system of three mutually perpendicular directions x, y, z, taken as reference directions. Geometrically, these can be represented as a triad of unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$, and a point in space is specified by the components (orthogonal projections) of its position vector onto the corresponding directions.

We can use an infinite variety of such triads. They are all distinct and at the same time can be obtained one from another by appropriate rotations and/or reflections.

A spherical coordinate system is determined by a radial distance of a particle from a reference point taken as the origin and its two angular coordinates – the polar and azimuthal angles (similar to latitude and longitude in geography). In this case, we also have an infinite variety of different systems of spherical coordinates – all associated with the same reference frame. They differ from one another by their position of origin, orientation of the polar axis, and the choice of the reference plain for azimuthal angle (similar to the choice of Greenwich meridian).

We see from these examples that a reference frame and a coordinate system are *different concepts*. And, in particular, the former does not specify the latter. We will often use in this book both terms, and it is important to distinguish between them. We will frequently abbreviate a reference frame as RF.

1.2
Weirdness of Light

The special theory of relativity has emerged from studies of electromagnetic phenomena. One of them is motion of light.

Let us extend our discussion of motions of physical bodies to situations involving light. Previously, we had come to the conclusion that one can catch up with any object. Does this statement include light? This question was torturing a high school student Albert Einstein more than a century ago and eventually brought him to special relativity. What we have just learned about velocity prompts immediately a positive answer to the question. Velocity is a relative quantity – it depends on reference frame. It can be changed by merely changing the reference frame. For instance, if an object is moving relative to Earth with a speed v, we can change this speed by boarding a vehicle moving in the same direction with a speed V. Then the speed of the object relative to us will be

$$v' = v - V. \tag{1.1}$$

We can change v' by "playing" with the vehicle – accelerating or decelerating it. For instance, reversing the speed of the vehicle would result in changing the sign of V in the above equation, and accordingly, would greatly increase the relative speed of the object without touching it. If we want to catch up with the object, we need to bring its relative velocity down to zero. We can do it by accelerating the vehicle to the speed $V = v$.

Because this works for objects like bullets, planes, or baseballs, people naturally believed that it should work for light as well. It is true that we never saw light at rest before. But, as an old Arabic saying has it, "if a mountain does not go to Mohammed, then Mohammed goes to the mountain." If we cannot stop the light on Earth, then we have to board a spaceship capable of moving relative to Earth as fast as light does and use this "vehicle" to transport us in the direction of light. Let c be the speed of light relative to Earth and V be the speed of a spaceship also relative to Earth. If Equation (1.1) is universal, then we can apply it to this situation and expect that the speed c' of light relative to the spaceship will decrease by the amount V:

$$c' = c - V. \tag{1.2}$$

Suppose that the rocket boosters accelerate the spaceship, its velocity V increases, and c' decreases. When V becomes equal to c, the speed c' becomes zero. In other words, light stops relative to us, that is, we have caught up with light. The same principle that has helped us "stop" the object in (1.1) at $V = v$, works here to help us catch the light. The law (1.1) of addition of velocities says that it is possible.

But there immediately follows an interesting conclusion. We know that the Earth can to a good approximation be considered as an inertial reference frame, and all inertial reference frames, according to mechanics, are equivalent. Einstein thought that this principle could be extended beyond mechanics to include all natural phenomena. If this is true, then whatever we can observe in one inertial system, can as well be observed in any other inertial system. If light can be stopped relative to at least one spaceship, then it can be brought to rest relative to any other inertial system, including Earth. In physics, if Mohammed can come to a mountain, the mountain can come to Mohammed. To stop light relative to the spaceship, we need to accelerate the ship up to the speed of light. To stop light relative to Earth, we may, for example, put a laser gun onto this ship and fire it backwards. Then the laser pulse, while leaving the ship with velocity c relative to it, will have zero velocity with respect to Earth. We then will witness a miraculous phenomenon of stopped light.

I can imagine an abstract from a science fiction story exploiting such a possibility, something running like this:

> *Mary stretched her arm cautiously and took the light into her hand.*
> *She felt its quivering wave-like texture, which was constantly*

changing in shape, brightness, and color. Its warm gleam has gradually penetrated her skin and permeated all her body, filling it with an ecstatic thrill. She suddenly felt a divine joy, as though a new glorious life was being conceived in her.

But, alas! Beautiful and tempting as it may seem, our conclusion that freely traveling light can be stopped relative to Earth or whatever else, is not confirmed by observation. It stands in flat contradiction with all known experiments involving light. As it had already been established before Einstein's birth, light is electromagnetic waves. The theory of electromagnetic phenomena, developed by J.C. Maxwell, shows a remarkable agreement with experiments. And both theory and experiments show quite counterintuitive and mysterious behavior of light: not only is it impossible to catch up with light, it is impossible even to change its speed in vacuum by a slightest degree – no matter what spaceship we board and in what direction or how fast it moves.

We have arrived at a deep puzzle. Light does not obey the law of addition of velocities expressed by Equation (1.1). The equation appears to be as fundamental as it is simple. And yet there must be something fundamentally wrong about it.

"Wait a minute!" the reader may say. "Equation (1.1) is based on a vast amount of precise experiments. It is therefore absolutely reliable, and it says that. . ."

"What it says is true for planes, bullets, planets, and all the objects moving much slower than light. But it is not true for light," – I answer.

"Well, look here: the speed of light as measured in experiments on Earth is about 300 000 km/s. Suppose a spaceship passes by me with the velocity of 200 000 km/s and I fire the laser pulse at the same moment in the same direction. Then one second later the laser pulse will be 300 000 km away from me, whereas the spaceship will be 200 000 km away. Is it correct?"

"Absolutely."

"Well, then it must be equally true that the distance between the spaceship and the pulse will be 100 000 km, which means that the laser pulse makes 100 000 km in one second relative to the spaceship. It is quite obvious!"

"Apparently obvious, but not true."

"How can that be?"

"This is a good question. The answer to it gives one the basic idea of what relativity is about. You will find the detailed explanations in the next chapter. It starts with the analysis of one of the most known experiments that have demonstrated mysterious behavior of light mentioned above. But to understand it better, let us first recall a simple problem from an Introductory Course of College Physics."

1.3
A Steamer in the Stream

The following is a textbook problem in nonrelativistic mechanics; however, its solution may be essential for understanding one of the experimental foundations of special relativity.

The problem is this: A steamer has a speed of u km/h relative to water. It starts at point A on the bank of the river with the stream velocity v km/h. It moves downstream to the point B on the same bank at a distance L from A, immediately turns back and moves upstream. How long will it take to make round-trip from A to B and back?

The solution is pretty simple. In the case of still water, the answer would be

$$t_0 = 2\frac{L}{u}. \tag{1.3}$$

Now, take account of the stream. If the steamer makes u km/h relative to water and the stream makes v km/h relative to the bank, then the steamer's velocity relative to the bank is $(u + v)$ km/h when downstream and $(u - v)$ km/h when upstream. We are interested in the resulting time, which is determined by the ratios of the distance to velocities. We must therefore use the speed averaged *over time*. The total time consists of two parts: one (t_{AB}), which is needed to move from A to B, and the other (t_{BA}) to move back from B to A. The time t_{BA} is always greater than t_{AB}, since the net velocity of the steamer is less during this time. Thus, the net velocity of the steamer is greater than u during the shorter time and less than u by the same amount during the longer time. Therefore, its average over the whole time is less than u. As a result, the total time itself must be greater than t_0. It must become ever greater as v gets closer to u. This result becomes self-evident when $v = u$. Then the steamer after turning back is carried down by the stream at the same rate as it makes its journey upstream. So it will just remain at rest relative to the bank at B and will never return to A. This is the same as to say that it will return to A in the infinite future, that is, the total time is infinite.

What if v becomes greater than u, that is, the stream is faster than the steamer? Then the steamer after the turn is even unable to remain at B; the stream will drag it down, taking it ever farther away from its destination. We can formally describe this situation by ascribing negative sign to the total time t.

Let us now solve the problem quantitatively. The time it takes to go from A to B and then from B to A is, respectively,

$$t_{AB} = \frac{L}{u+v}, \quad t_{BA} = \frac{L}{u-v}. \tag{1.4}$$

So the total time

$$t_{\downarrow\uparrow} = t_{AB} + t_{BA} = \frac{L}{u+v} + \frac{L}{u-v} = \frac{t_0}{1-(v^2/u^2)}, \tag{1.5}$$

where t_0 is the would-be time in the still water, given by Equation (1.3).

If we plot the dependence (1.5) of time versus stream velocity, we get the graph shown in Figure 1.5.

Equation (1.5) describes in one line all that was written over the whole page and, moreover, it provides us with the exact numerical answer for each possible situation. The graph in Figure 1.5 describes all possible situations visually. You see that for all $v < u$ the time t is greater than t_0, it becomes infinite at $v = u$, and negative at all $v > u$. When v is very small relative to u, Equation (1.5) gives $t_{\downarrow\uparrow} \approx t_0$. This is natural, since

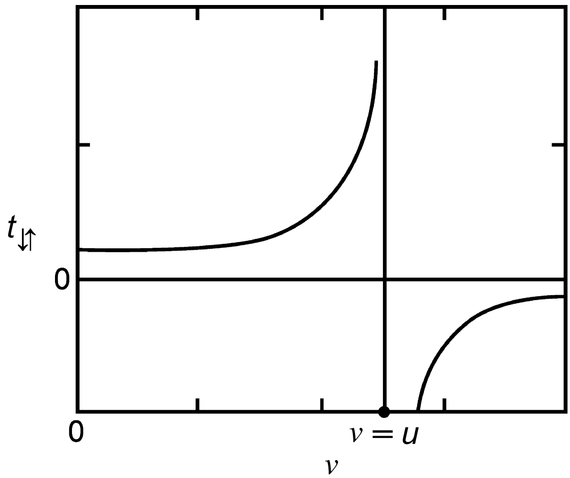

Figure 1.5 The dependence of round-trip time $t_{\downarrow\uparrow}$ on speed v.

for small v the impact of the stream is negligible, and we recover the result (1.3) obtained for the lake.

Now, consider another case. The river is L km wide. The same steamer has to cross it from A to B right opposite A on another bank and then come back, so the total distance to swim relative to the banks is again $2L$. How long will it take to do this?

The only thing we have to know to get the answer is the speed of the steamer u' in the direction AB right across the river. The steamer must head all the time a bit upstream relative to this direction to compensate for the drift caused by the stream. If during the crossing time the steamer has drifted l km downstream, then to get to B, it must head to a point B′, l km upstream of B. Thus, its velocity relative to water is u and directed along AB′, the velocity of the stream is v and directed along B′B, and the resulting sought-for velocity of the steamer relative to the banks is directed along AB. These three velocities form the right triangle (Figure 1.6), and therefore

$$u' = \sqrt{u^2 - v^2} = u\sqrt{1 - (v^2/u^2)}. \tag{1.6}$$

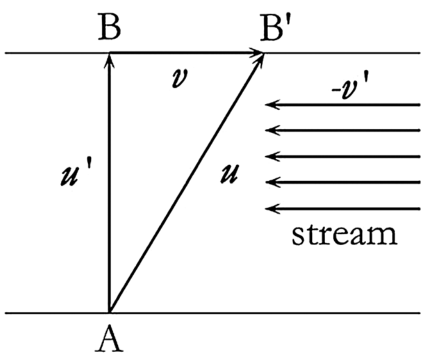

Figure 1.6

So our final answer for the total time back and forth between A and B is

$$t_{\perp} = \frac{2L}{u'} = \frac{2L}{u\sqrt{1-(v^2/u^2)}} = \frac{t_0}{\sqrt{1-(v^2/u^2)}}. \tag{1.7}$$

Note that Equations (1.6) and (1.7) give a meaningful result only when $v < u$ (a side of the right triangle is shorter than hypotenuse). Then, according to (1.7), time $t_{\perp}$ is also greater than t_0, but it is less than $t_{\downarrow\uparrow}$. Thus, one can write

$$t_0 < t_{\perp} < t_{\downarrow\uparrow}. \tag{1.8}$$

If $v > u$, the triangle in Figure 1.6 cannot be formed. The steamer's drift per unit time exceeds its velocity u and the steamer would not be able to reach the point B, let alone return to A. This circumstance is reflected in the mathematical structure of Equations (1.6) and (1.7), which yield imaginary numbers when $v > u$. They say that there is in this case no physical solution that would satisfy the conditions of the problem.

Now, what is the link between this problem and the experiment with light mentioned above? Take the running waves on the water surface instead of the steamer and you turn the mechanical problem into the hydrodynamic one. Then take the sound waves in air during the wind instead of the steamer on the water stream and you get the same problem in fluid dynamics. And as the last step, consider the light that propagates in a moving transparent medium in transverse and longitudinal directions and here you are with the optical problem that is identical to the initial mechanical one.

This is why I started the book with this *introductory physics* problem. Its mathematical description is exactly the same as that of the problem ahead. But keep in mind that the treatment of this "sample problem" is nonrelativistic! This is totally OK for a steamer or for the sound in air; the corresponding errors in results obtained are quite negligible in these cases, so we can use them safely. But when we apply the same to the case of light, the result will be totally erroneous. It was precisely this wide discrepancy between the nonrelativistic prediction and actual observation that became one of the crucial scientific evidence in favor of Einstein's theory.

Problems

1.1 A steamer has a speed u relative to water. It started from A, went downstream, reached B on the same bank at distance L, immediately turned back and returned to A. The recorded round-trip time of this travel was t. Find the speed V of the stream in terms of u, L, and t.

1.2 With the same data as in the previous problem, find the speed of the stream for the case, when the points A and B lie exactly opposite each other across the river.

1.3 For the same data as in Problem 1.1, find the average speed of the steamer over time t and over distance $2L$. Which average is greater?