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Mathematics, Mysticism, and More
From Pythagoras and Plato to Pauli

"All is number."

Pythagoras

"I confess, that very different from you, I do find sometimes scientific inspiration in mysticism ... but this is counterbalanced by an immediate sense for mathematics."

Pauli, in letter to Bohr

The unreasonable efficiency of mathematics in science is a gift we neither understand nor deserve.

Wigner

Wolfgang Pauli (Nobel Prize 1945) kicks a ball toward the camera of Roy Glauber (Nobel Prize 2005).

The Demon and the Quantum: From the Pythagorean Mystics to Maxwell's Demon and Quantum Mystery. Robert J. Scully and Marlan O. Scully

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1.1 The Pythagoreans[1)]

Young Pythagoras was hungry, cold, and discouraged after his third night at the temple gate. His teacher Thales had taught him geometry and then sent him off to Egypt. He had been warned that entrance into the temple was no small matter, and it was not, for he had been turned away at temple after temple. At least here, they had not turned him away. "Wait." That is all the priest had said, his tone having as much emotion as did the yellowish desert sand that surrounded him. Through three days of blood-boiling heat and three nights of bone-chilling cold, he was tormented by blowing sand, its gritty texture wearing away his hope and spirits.

Just as he was about to give up, Pythagoras was finally invited to enter the temple. There, at the hands of his Egyptian masters, he learned math and mysticism, from the years 535 to 525 BC. After a decade or so, he was captured by the king of Persia, Cambyses II, who invaded Egypt around 525 BC. Iamblichus writes that:

> "[Pythagoras] was transported [to Persia] by the followers of Cambyses as a prisoner of war. Whilst he was there he gladly associated with the Magoi ... and was instructed in their sacred arts and learnt about a very mystical worship of the gods."

Some seven years later finds Pythagoras in southern Italy in the city of Croton, where he founded his famous school. There he created what is perhaps the first system of higher education that left a permanent mark on history. The philosophical–theological school of Pythagoras was unique. The story of his community has survived the eroding effects of two and a half millennia. As with many historical records, this one lacks color and vitality, yet the Pythagoreans offer an accurate and detailed account of the procedures governing their approach to higher education. Students accepted by Pythagoras took a vow of loyalty binding them to him and to their fellow students. The selection process was rigorous and demanding, much as the one Pythagoras himself had experienced in Egypt.

Many aspects of the Pythagorean school can still be seen in today's universities. His educational model had been molded by what he had experienced in Egypt, in his time the highest center of learning in the world. Before being admitted to the Egyptian institution, he first had to be examined by the elders. When accepted, he then had to pass an initiation, of which little is known. Thus, we have some of the most obvious earmarks of elite education as it persists to this day: highly selective, the desirability (real or perceived) of traveling abroad to get a broader education, as well as secret initiation practices. To clarify the term "secret" we note such initiation practices still survive in various fraternities/sororities today. Secrecy equals exclusivity, which in turn tends to generate interest and even some aspects of respect.

Because he completed his studies with the Egyptians and Persians, Pythagoras was respected and valued by society. Upon his return to Greece he had little difficulty in establishing a type of college loosely modeled on the one he himself had attended. More than just a college, it was also a community, in some respects similar to a monastery. Members' food and dress were simple and their discipline severe. One joined the Pythagorean community for life by surrendering all earthly possessions and totally submitting to the new system. It must have been worth the sacrifice, for there is no record of dissention among the initiates. The Pythagoreans were a self-supporting group and relied upon a system of communal property. They were also one of the first elite groups to recognize women as equals.

Pythagoras was the first to realize and to teach that proof proceeds from assumptions. The theorem bearing his name (see Fig. 1.1) may or may not have been his discovery, but he was clearly the first to introduce the idea of "proof" into mathematics.

The Pythagoreans were fascinated by the relation of number to musical tone. They determined the tone of a musical instrument was governed by the length of its strings. Decreasing the length of the strings or increasing the tension increases the pitch. This gives rise to the explanation of the physics of sound waves, another beautiful example of nature described by mathematical rules (Fig. 1.2).

The physics that applies to sound waves is similar to the physics employed in other wave phenomena. The waves have a frequency, measured in hertz, which is the number of up and down wave pulsations per second. The human ear is sensitive to frequencies between 20 and 20,000 hertz. The Greeks developed notes carrying six different frequencies on the handmade flutes they played (today's musical instruments employ 12 notes). The Pythagoreans thus described frequencies with the use of simple ratios. This inter-

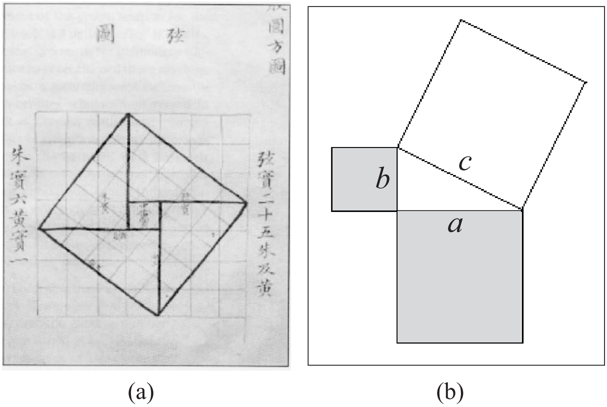

Fig. 1.1 (a) Chinese and (b) Greek approaches to the famous Pythagorean theorem, $c^2 = a^2 + b^2$.

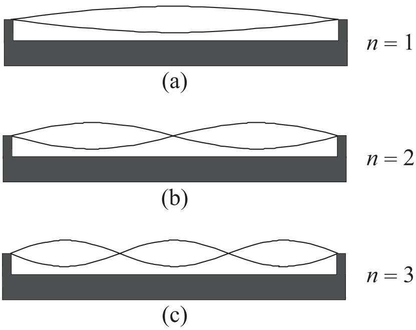

Fig. 1.2 The harmonies of a monochord. When a taut string (called a monochord) is plucked we see, and hear, first a note corresponding to vibration as a whole. This situation is denoted in (a). Then it vibrates in two parts, then three parts, as in (b) and (c), and so on. In this way, the tonal harmonies are seen to be described by numbers (a) $\rightarrow$ 1, (b) $\rightarrow$ 2, (c) $\rightarrow$ 3, etc.

esting interplay between math and music had a profound impact on the Pythagoreans.

The concept of math as an overarching theme intertwined with science, astronomy, and engineering is understandable, but why music? As the Pythagoreans proved, music *is* mathematical, and subjects with a mathematical foundation seem to become more and more analytical over the years.[2] The necessary mathematical rhythm in music is met when the chords involve frequencies that are multiples of each other. For example, if we play a chord at 440 hertz, then the other chords which will harmonize with it are those with frequencies of 880 hertz, 1320 hertz and 1760 hertz. But if we instead used chords of 600 hertz, 900 hertz and 2000 hertz, the harmonics would be lost and the listeners would leave the theater.[3] Science has yet to fully explain why the human ear enjoys this mathematical mixing of harmonics.

In an interesting parallel, radio antennas, as electromagnetic receptors, bear a striking similarity to the music preferred by our ears. If an antenna is broken, it will receive certain frequencies better if you break it at half its original length. These are obviously ratios necessary to produce harmonics like those used in music.

From music to astronomy, the Pythagoreans saw numbers as the answer to all questions. For this and other reasons, the Pythagoreans were fascinated with the intriguing realities involved in mathematical computations of any kind. In the harmony of mathematics they felt they had discovered an underlying form of consciousness and intelligence; in short, they thought they had found a path to god. More correctly, they had discovered their own chosen vision of god. Pythagoras was intrigued by such questions. His statement summarizes his philosophy:

> "Reason is immortal, all else mortal."

The clearest, deepest form of reason is mathematics, and numbers are at the heart of mathematics. Therefore, it is understandable that Pythagoras felt that "God was number." At least he felt that "number" was the best path to his god.

"All is number" represents a philosophical reckoning of the universe by the Pythagoreans. In context, the early Greeks believed that the universe was made of four elements: air, earth, fire, and water. A similar attitude is voiced by the ancient Chinese, who believed that five elements made up the whole universe: metal, wood, water, fire, and earth.

The mathematical discoveries of the Pythagoreans were as logical as anything we are developing today. Parts of their philosophy were misguided, but many of their contributions are lasting. In fact, their discoveries are often what are taught today. Their analytical approach to geometry, their conviction that number is the key to the nature of the universe, and their belief in a communal life in service of religion are original Pythagorean concepts, ideas that still attract modern adherents.

There is at least one newer argument for the idea of "God" as number. IBM physicist Rolf Landauer made the point that information is just as real as electricity, water, rocks, and the wind. In this book, we will investigate this idea from several different points of view. We will reinforce the idea that information is a fundamental, "real" quantity, encountered in thermodynamics, computer science, and quantum mechanics. Even without such demonstration, we can see a connection between information and number.

Let us consider the state of a computer as nothing other than a bunch of numbers. Every element of "yes" or "no" memory is given by a simple number. Let us agree that the number is "1" for "yes" and "0" for "no." Mathematicians call this a binary number. At any given moment, we specify the state of the computer by a very large array of numbers, positive ones or zeros.

It is not a large step from there to think of our brain as a computer, and the eminent John von Neumann made this connection in his writings. The same argument would say that we are, at any instant, representable as a number. Such thinking once was strictly within the domain of philosophers, yet as we uncover more of nature's secrets, it is becoming more and more a part of working science. Perhaps the crazy Pythagorean notion of nature-as-number is easier to fathom today than in earlier times.

1.2 The Mind Expanders -1, $\sqrt{-1}$ and Ψ

"Minus times minus is plus for reasons we will not discuss."
W. H. Auden

Not surprisingly, mathematics, the science we use most in our daily lives, was the first science to be developed. All ancient societies developed mathematics. They began by using hieroglyphics and ingenious sequences of coded dots and dashes to facilitate highly adept forms of computation. These systems were used to aid in the construction of the massive building projects of antiquity's great cultures. The ability to pull off such engineering marvels at very early stages of cultural development shows much about the mathematical abilities innate to man.

The wonders of the world were often the mother of invention in the development of math. The ancient Babylonians relied upon a series of irrigation canals requiring many workers who needed to be paid, thus requiring a system of payment and record keeping. The Egyptians and the Mayans were very clever in the application of mathematics in the construction of their pyramids. During their golden age, the Indians explored the mysteries of π and contributed much to the understanding of algebra.

We are forced to use our imagination to comprehend their accomplishments, for too few records have survived to document the development of math. Yet, the history of math is a case study of the most frequently employed applications of pure science known to man. This history shows us a great deal about where we have come from and where we are bound. Fortunately, something of the story of math survived. For instance, we know the ancients were intrigued by π,[4] an irrational number (i.e., a number which cannot be expressed as the ratio of two whole numbers), just as they were fascinated by the circle and its intimate relation to π. They were somewhat philosophical in their thinking, emphasizing that the circle itself resembled π in having no stopping point. People in China, India, and Japan were particularly determined to compute this riddle number. In 264 AD, one Liu Hui constructed a polygon of 3072 sides and from this model calculated π to be 3.14159.

Unlike the number systems of other societies, which have been lost, forgotten, rediscovered, and rewritten, the Chinese have always used a decimal system in mathematics, and discovered the concept of zero early on. Indeed, the use of zero and negative numbers marks a watershed in mathematics. In today's credit card, debt-ridden society, negative numbers are not in the least abstract. Being broke (zero cash) is one thing. Being in the hole with a big debt hanging over our head imposes a painful dramatization of a negative quantity. When we are in debt, we have to pump cash into the system just to get back to broke. Negative numbers can be painfully real.

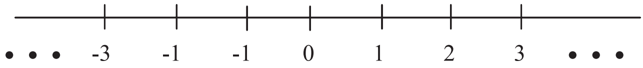

Fig. 1.3 The numbers, positive and negative, define positions on a line.

Another example of a negative quantity can be perceived by traveling in the wrong direction from one's destination. Thus, our velocity along the highway could be said to be $+60$ mph when we are moving toward our destination. However, if we make a mistake and are going in the wrong direction, then that is properly denoted as -60 mph.

In general, we draw a line and call the numbers to the right of zero positive. Those to the left are negative, as in Fig. 1.3.

As per the quote at the beginning of this section, minus times minus is plus. We perhaps recall this from our high school algebra. But why does minus times minus produce plus? What is the deal? Think of it this way: Suppose we are headed home at 20 mph and we double the velocity to 40 mph; this is just 2×20 mph. So far so good; now let's suppose we are going home at 20 mph and we flip a U-turn. Now, our velocity is clearly -20 mph, which is $(-1) \times 20$ mph. Then, if we flip another U-turn, our velocity is now $(-1) \times (-1) \times 20$ mph, and since we are clearly headed home again, we must be going $+20$ mph. Therefore, $(-1) \times (-1) \times 20$ mph $= +20$ mph, so:

"Minus times minus is plus for reasons clear to all of us."

R. J. Scully

Meanwhile, back at Pythagoras Polytech, things are not going so well. They have just learned that $\sqrt{2}$ is a strange (irrational) number, not expressible as the ratio of whole numbers. No ratio of two integers, no matter how complicated, will give the square root of 2. This alarming discovery really shook the faith of the Pythagoreans. If God is number, then how can a simple thing like $\sqrt{2}$ be imperfect?

But the square root of 2 is "imperfect" (irrational) and, consternated by this realization, Pythagoras's followers were pledged to secrecy. One of them (by the name of Hippasus, according to tradition) talked too much about the dirty $\sqrt{2}$ secret and, as the story goes, was drowned by his colleagues.

Just imagine if they had found out about negative numbers! More to the point, what if they had learned about $\sqrt{-1}$? Not only is there no rational number for $\sqrt{-1}$, there is no real number which multiplied by itself will give -1. In fact, $\sqrt{-1}$ is called "i," which stands for imaginary. This would really have had the Pythagoreans ready to mutiny.

But fast forward a couple of millennia to the time of the quantum. In quantum mechanics we use $\sqrt{-1}$ to represent nature. You can hear the Pythagoreans say, "This is just surreal." It is too mystical even for them! That is what science is all about – one of the messages of this book is that, however weird the mystics were, however strange their view of nature, the real world is even stranger than they (or we) would ever have imagined.

1.3 Let *x* Be the Great Unknown

"Al-jebr: reuniting broken bones."

Tradition has it that the Arabs brought algebra, alchemy (the forerunner of chemistry), and alfalfa to the world as they made their push westward. "Al" is Arabic for the definite article "the," thus alfalfa is "the grass." The word alchemy comes from the combination of Greek theory and Egyptian practical knowledge of materials, e.g., in mummification. This early fusion of Egyptian and Greek science was called "khemia" and the Arabs called it "alkhemia" or "alchemy."

Therefore, I always assumed that algebra stood for something like "the calculation" or "the arithmetic plus logic." Not so. According to the dictionary, the Arabic root is "Al-jebr," referring to the process of reuniting, as in the setting of bones. So the word "algebra" actually means something like "the method of unification," as in uniting or setting things equal. Anyway, those who claim algebra broke their bones will be interested to know that algebra is really more like resetting broken bones.

A little algebra makes a big difference in our ability to solve problems in modern science and technology. Algebra gets results quickly and accurately. According to my childhood encyclopedia, the modern-day high-school freshman knows more algebra than the wisest of the ancients 4000 years ago.

Simple algebra is the next math milestone after arithmetic and geometry. Arithmetic has proved indispensably useful, algebra even more so. A fair comparison would be using our fingers to dig in the garden (arithmetic) compared with using a shovel (algebra). For these reasons, we will present (and use) a little algebra in the Endnotes. Time spent there will be richly rewarded – try it, you will like it (and use it).

In particular, the use of exponential notation and logarithms is useful and important. For example, the number 100 can be written as 10×10, i.e., 10 squared, which we can write as 10^2. Likewise 1000 is 10^3 and one million is 10^6, etc. Any number can be written as 10 to some power or number. The actual power is called the logarithm of that number. For example, the logarithm of $100 = 10^2$ is 2 and the logarithm of 10^6 is 6, etc. Let me raise a flag here. The ideas of exponents and logarithms are very important. If these ideas are new to you, do not worry; they are simple – make them your friends. To learn more, please turn to Endnotes 1.

1.4 Math, Mind and Mankind

There are many similarities in the development of math among different cultures. Throughout time, we observe among various peoples insights into the workings of the human mind, and we gain thereby testimony to the mind's aptitude for working with numbers and geometrical quantities. How that aptitude has been developed is parallel to the manner in which societies developed. Unlike other forms of history, scientific history has the advantage that what is true and time-tested tends to be generally adopted, unlike political history, where the victor writes the history. Furthermore, the creators of science, as such, are competent. The facts always (eventually) win out. Nothing is more nearly perfect than math; no account of scientific discovery gains the day until reduced to number. A brief overview of math and number reveals the obvious as well as encourages more subtle inferences:

1. Math ultimately wins a hearing despite differing ideologies. As we have noted, the Pythagoreans tried to suppress the fact that $\sqrt{2}$ is an irrational number, but failed.
2. Early civilizations required some form of math in order to foster technological development, e.g., to keep track of inventories. Yet the connection between math and the mind goes deeper. Math begins to glimmer even in newborns. As we will discuss later, it would seem that humanity is born hardwired for math.
3. Societies on the rise develop and propagate math. Societies on the decline neglect education in general and mathematics in particular.

The last two statements call for further discussion. One might argue that this math–mind kinship affords evolutionary advantages. Successful hunting and protection of the clan require cooperation, organization, and logical planning. Mathematical thinking greatly assists activities vital to survival. Research into the nature of man and mathematical proficiency has been reported by Brian Butterworth[5] in his book *What Counts*. Butterworth found that mathematics might have been in use for tens of thousands of years, since he observed that cave drawings and Neanderthal relics often show the hash marks of the dot–dash method that was early man's usual form of numerical notation. He comments on research into the reactions of newborns to dot and dash configurations. Thinking patterns have been inferred in newborns by making observations of their stares and glances when shown pictures of objects. When shown cards displaying a series of dots and dashes, the duration of the infants' stares can be predicted by the number of dots and dashes with which the babies are confronted. Twice as many dots produce a stare twice the duration. Not that twice as many dots are twice as interesting. Rather, this evidence of infantile "counting" provides evidence for the innate mathematical orientation of human beings. Butterworth focuses also upon neurological studies of math prowess, a dauntingly complex, but nonetheless verifiable, phenomenon apparently inseparable from human nature.

Scientists like Butterworth are realizing that the teaching of numerical problems should be reformed to hasten and sharpen understanding. Introducing math to young minds can be substantially facilitated when the naturally hardwired talents of the human mind are taken into consideration. At present some members of the teaching establishment believe the calculator is a poor substitute for mental exercise; they argue that the device results in our declining to a nation of math illiterates who don't even know the multiplication tables.

We are currently repeating an old failure. As our schools decline, math is usually the first subject to be neglected. Consequently, the average American teenager displays a mathematical proficiency much below that of their Chinese counterparts. Collateral damage follows, with neglect of opportunities for the individual to develop reasoning and judgment skills. You cannot fast-talk your way to success in mathematics – answers are right or wrong. This pattern of neglect is not new, for throughout history math has been developing and advancing only later to be neglected for centuries. The past few decades of Western education have yielded mathematical backsliding.

After a few hundred years of advancing in mathematics, we are now beginning to slip. In the past, numerous obstacles hampered mathematics in the Western world. This continued until approximately the beginning of the 17th century. For example, measurements were imposed by the writs of kings, resulting in many absurd measurement systems that still linger in some regions. France, for example, had 2000 different units of measure in 1788. There was no uniformity of measurement from one city to another. This diversity was unnecessary. As much as one thousand years earlier far fewer and more reasonable standard measures had been adopted by King Charlemagne. Regardless of its other accomplishments, society can never become fully effective in science and technology if it lacks a solid mathematical base.

Is it possible that today we are underrating the importance of the first science discovered? If so, we should learn from the lessons of such negligence in the past. As noted by Beckman, the following words of the ancient Egyptian scribe, Ahmes, offer appropriate guidance as to what the foundation of true science is:

> "Accurate reckoning. The entrance into the knowledge of all existing things and all obscure secrets."

What then has the work of the Pythagoreans brought us? It is another new piece of reality. Number is as real as air, earth, fire, and water, even though it is abstract. It is *very* useful and, as we will see in subsequent chapters, can be applied to what is real and knowable.

1.5 Plato – Another Kind of Pythagorean

Another tangible legacy of the Pythagorean school is its impact on later Greek scholars such as Plato. Indeed, the thoughts and insights of Plato deeply influenced quantum pioneers Heisenberg, Planck, and Schrödinger. According to biographer Armin Hermann[6)]

> "In the course of his life, Heisenberg assimilated a great many intellectual ideas, and the greatest influence of all was platonic philosophy."

So who was this man Plato? Among other things, he was an extension of the Pythagorean theme. However, he was more of a philosopher than a Pythagorean. To him, "ideas" were the most real and valued entities in the universe.

Plato clearly learned from and was influenced by the teachings of the Pythagorean school, but Plato did not hold their philosophy of "number as all." However, he certainly adopted their views on much else. Plato saw in shapes, forms, planets, music, even reality and existence, the constantly guiding force of mathematical reason. This would become the theme of his own school, with its entrance door marked with a sign,[7)]

> "Let no one ignorant of mathematics enter here."

Like many other philosophers, Plato felt that mathematics was the finest training possible for the mind.

As we shall see in later chapters, quantum physics seems to hold some sympathy for the views of Plato. For example, in the dialog *Timaeus*, Plato makes an interesting observation, which is very much in harmony with modern physics:

> "If in a discussion of many matters ... we are not able to give perfectly exact and self-consistent accounts, do not be surprised: rather we would be content if we provide accounts that are second to none in probability."

The quantum mechanic could not agree more. Quantum mechanics is probabilistic. Yet Plato does not think nature is totally random or chaotic. On the contrary, he stresses the role of an overarching intelligence in the universe. Plato, like the Pythagoreans, envisioned a perfect, mathematically elegant, universe. The fact that we often use a mathematical approach in everyday science and technology deepens our appreciation for and debt to the Pythagoreans.

1.6 Pauli – a Latter Day Pythagorean

It is fascinating that the foundation of quantum physics, one of the newest and most advanced fields of science, has aspects in common with the philosophy of Plato. Indeed, Plato finds an ally in the physicist Wolfgang Pauli (1900–1958), often regarded as the conscience of quantum physics. Pauli was one of the fathers of quantum physics as well as being a significant contributor to subjects ranging from Einstein's theory of relativity to the periodic table. Pauli is also regarded as the junk-yard-dog of quantum mechanics.

He was himself a prime mover in the formulation and application of quantum theory. For example, the Pauli exclusion principle explains how electrons conspire to make up atoms with only one electron per state. Some 15 years later he deepened our understanding of the behavior of electrons by using more advanced quantum mechanics (i.e., the quantum theory of fields) to show the relation between the exclusion principle and the internal properties (spin) of the electron. This is vintage Pauli – always looking for better, deeper understanding of nature and her ways.

Pauli was born in 1900 in Vienna, where his father, Wolfgang Joseph Pauli (WJP), was a professor of chemistry. Although Pauli's father was from a Jewish family in Prague, he (WJP) changed his name and converted to Catholicism. Wolfgang Pauli was baptized into the Catholic church and the famous Ernst Mach, whom we shall meet again later, was his godfather. His outstanding promise as a budding mathematician became apparent in his early years, and his father hired special tutors to train him in mathematics. He attended the university of Munich, earning his Ph.D. in 1921 under the leading light, Arnold Sommerfeld.

It is interesting to note that, although Pauli was known for his sharp wit and brusque manner, he was particularly deferential to his thesis father, Sommerfeld. After getting his Ph.D., he went to work for Max Born and then Niels Bohr, but he always treated Sommerfeld as *the* "geheimrat" professor. Around 1924, he developed the Pauli exclusion principle, mentioned earlier, and obtained a teaching position at the University of Hamburg. In the late 1920s, he moved on to the ETH in Zurich, which is the MIT of Europe. Except for a few years during World War II, when he was, among other things, "dean" of theoretical physics at Princeton, he spent most of his life in Zurich.

Pauli had many students and assistants who later became famous, for example, Victor Weisskopf, who was to become one of the most beloved, leading figures in physics. Weisskopf describes Pauli as being without guile and without regard for persons. He treated everyone alike: "big shots and small shots" got the needle equally from Pauli. He was not personal or vindictive, he just disliked sloppy thinking and half-baked ideas. As a result he would often criticize people in public: "This is stupid, go home and work it out" was a typical Pauli seminar criticism.

He was famous for his one-liners. For example, when a colleague (Paul Scherrer) presented an idea to Pauli and exclaimed, "Isn't that simple?", Pauli responded: "Simple it is, but it is also wrong." Another time when the University of Rochester asked for a recommendation for Weisskopf as professor of physics, Pauli responded: "I have nothing against this man." Everyone knew that was very high praise coming from Pauli and Weisskopf got the job.

Later in his career Pauli became interested in philosophy and mysticism. He collaborated with the famous psychiatrist and psychotherapist Carl Jung. Pauli was one of his favorite patients, and Jung made an elaborate analysis of more than four hundred of

Pauli's dreams. The psychological researchers of the day were in a different world from that of the hard sciences. Basically, however, the physical and psychological scientists were both trying to gain an understanding of the role of consciousness in human behavior. Their viewpoint was not unlike Plato's philosophy of reality and existence. They also had interest in the baffling questions posed by the role of the observer and the observed in quantum physics. But modern physical science demands experiment and proof. So Pauli made public very little of his research in this field, choosing instead to communicate his ideas through letters to colleagues and working in person with Jung.

This was, in some ways, similar to his mainstream scientific career, in which he published little, choosing instead to advance science by writing down his new ideas in letters that he sent to others like Heisenberg and Schrödinger. He wrote many thousands of these and it is likely that many findings that should have been credited to him went instead to others. But, no one ever doubted his genius. As Max Born said, ''Since the time when he was my assistant in Göttingen, I knew he was a genius comparable only to Einstein himself. As a scientist he was, perhaps, even greater than Einstein. But he was a completely different type of man, who, in my eyes, did not attain Einstein's greatness.'' By age 21 he had finished his Ph.D., publishing a 237 page article on Einstein's theory of relativity. Einstein himself had nothing but praise for the work and it is still one of the most frequently used references to relativity today.

He never lost interest in the interplay between observer and the observed in quantum mechanics. Rather, his interest grew stronger and more philosophical. Below is a quote from one of his letters to Niels Bohr written in 1955:

> ''What really matters for me is . . . the more active role of the observer in quantum physics . . . According to quantum physics the observer has indeed a new relation to the physical events around him in comparison with the classical observer, who is merely a spectator.''

In another quote from the same letter (a part of which is at the beginning of this chapter), he expounds upon his thoughts on mysticism:

> "I confess, that very different from you, I do find sometimes scientific inspiration in *mysticism* (if you believe that I am in danger, please let me know), but this is counterbalanced by an immediate sense for mathematics. The result of both seems to be my kind of physics, whilst I consider epistemology merely as a logical comment to the application of mathematics in physics."

As can be inferred from this quote, Pauli found that the conventional, scientific approach to nature is only one possibility. There are other approaches, such as the intuitive approach, which seems to be the antithesis of science but often provides valuable hints to solving problems. In this sense, Pauli saw logic and intuition as being two different (but not contradictory) approaches to problems.

Pauli won the Nobel Prize for physics in 1945 for the exclusion principle. He spent most of his life in Zurich, with a six-year period in Princeton as professor of theoretical physics during World War II. He left when Austria was annexed by the Nazis but returned to Europe after the war, settling in Zurich. Pauli was awarded the Max Planck medal in 1958 and in that year was later hospitalized with pancreatic cancer. When his assistant came to visit, Pauli asked him if he had noticed that he was in room 137. Throughout his life he had sought an answer to why the dimensionless fundamental constant involving Planck's quantum of action, the charge on the electron, and the speed of light, has the value of almost exactly 1/137. This important number in physics brings together electricity, relativity, and quantum mechanics. In that Zurich hospital on December 15, 1958, Pauli died in room 137.

Key Points

- Algebra unites.
- Exponential notation is neat, e.g., one thousand $= 10^3$.
- Logarithms are also cool, e.g., $\log_{10}(10^3) = 3$.
- Plato was essentially a Pythagorean whose ideas seem closer to those of a modern quantum physicist like Pauli than you would have ever guessed.