1
Characteristics of Low-Temperature Plasmas Under Nonthermal Conditions – A Short Summary

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1.1
Introduction

The concept of a plasma dates back to Langmuir (1928) and originates from the fundamental difference between regions of electrical gas discharges which are distant from boundaries (bulk of the discharge) compared with regions which adjoin the boundaries (sheaths).

1.1.1
Definition

Plasmas are quasineutral particle systems in the form of gaseous or fluid-like mixtures of *free* electrons and ions, frequently also containing neutral particles (atoms, molecules), with a large mean kinetic energy of the electrons and/or all of the plasma components (0.2 eV to 2 MeV per particle) and a substantial influence of the charge carriers and their electromagnetic interaction on the system properties.

The interactions between the electric charges of the plasma components show two aspects:

- Coulomb interaction among the charge carriers. Owing to the long range of the Coulomb force in the case of large charge-carrier densities ($n_e \gg 1/\lambda_D^3$), each charge carrier interacts simultaneously with many others (collective interaction).
- Formation of macroscopic space charges (in the frame of quasineutrality) as a consequence of external influences and modification of charge-carrier movement in the electrical field of these space charges.

Related to the quasineutrality and the presence of free charge carriers, the most intrinsic attribute of the plasma state is its tendency to minimize external electric and magnetic fields inside the bulk, in contrast to its behavior in the surrounding sheaths.

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Table 1.1 Subdivision of plasmas.

Low-temperature plasma (LTP)		High-temperature plasma (HTP)
Thermal LTP	Nonthermal LTP	
$T_e \approx T_i \approx T \lesssim 2 \times 10^4$ K	$T_i \approx T \approx 300$ K $T_i \ll T_e \lesssim 10^5$ K	$T_i \approx T_e \gtrsim 10^7$ K
e.g., arc plasma at normal pressure	e.g., low-pressure glow discharge	e.g., fusion plasmas

1.1.2 Types of Plasmas

Plasmas are frequently subdivided into low- (LTP) and high-temperature plasmas (HTP). A further subdivision relates to thermal and nonthermal plasmas (see Table 1.1).

1.2 Starting Point for Modeling the Plasma State

There are three different basic approaches toward a theoretical description of the many-particle plasma state: single-particle trajectories, kinetic and statistical theory, and hydrodynamic approximation.

1.2.1 Single-Particle Trajectories

This model is based on the motion of individual particles (e.g., under the influence of the Lorentz force). Problem: The electric and magnetic fields in the plasma must be regarded as given and cannot be obtained in a self-consistent manner from the cooperative movement of the particles. Using Monte Carlo simulations the study of single-particle trajectories could be extended to kinetic ensembles taking into account the effect of collisions. This technique is an alternative to the kinetic theory.

1.2.2 Kinetic and Statistical Theory

On the basis of kinetic criteria each particle ensemble of the plasma is analyzed taking into consideration the specific conditions and generalizing the kinetic theory of neutral gases to plasmas. The ultimate goal is to be able to calculate the space and time dependence of all the interesting distribution functions by solving the kinetic equations.

For nonthermal low-temperature plasmas the most important of these is Boltzmann's equation (1872) for the energy or velocity distribution of the electron component. This equation describes the balance of the particle density in phase space. The total time derivative of the distribution function is the sole outcome of particle collisions, contained in the so-called collision integral, which usually encloses a multitude of terms for different collisions of electrons (elastic, inelastic, collective, etc.). After explicit replacement of the external forces by the general Lorentz force, which then appears as self-consistently given by electric and magnetic fields of space charges and moving charge carriers, the collision-free approximation of the Boltzmann equation reduces to the Vlasov equation.

The kinetic theory is the strongest instrument of plasma theory, e.g., for handling extreme nonequilibrium conditions as well as deviations from the Maxwell distribution function in many realistic plasmas.

1.2.3
Hydrodynamic Approximation

This model treats the plasma as a continuum and determines the interesting macroscopic characteristics (density, flow, pressure, etc.) from the balance equations of the number, energy, and momentum of each particle species. The balance equations are obtained as integrals of the appropriate kinetic equation. A special form of this approximation is the so-called magneto-hydrodynamics (MHD) model, in which the plasma is considered as an electrically conducting liquid under the influence of magnetic fields.

1.3
The Role of Charge Carriers

The existence of charge carriers as the dominating components of the plasma is connected with a series of characteristics which are also important in industrial applications. The most active component of a nonthermal low-temperature plasma (LTP) is the hot electron gas. The high mean kinetic energy of electrons results in the generation of electromagnetic radiation (lines and continua) and in the production of numerous ionized, excited, and dissociated species of increased chemical activity. Applications are plasma light sources and plasma chemical reactors.

Furthermore, the existence of charge carriers manifests itself in the following:

- occurrence of electrical conductivity,
- screening of electric fields,
- occurrence of a multitude of oscillations and waves, typical for the plasma (Langmuir oscillations, ion acoustic oscillations, cyclotron oscillations, drift waves, surface waves, Alfven waves, etc.), as well as corresponding instabilities (plasma turbulence),
- interaction with magnetic fields. This is an important aspect of modern plasma physics. In the interaction with magnetic fields the whole spectrum and variety of the typical plasma properties become effective, and
- formation of characteristic boundary sheaths due to the contact of plasmas with solid surfaces. This is of particular importance in the technology of plasma processing.

1.4
Facts and Formulas

1.4.1
Electron Energy Distribution Functions (EEDF)

Calculation of the distribution functions F for the velocity or energy of the electron component under existing conditions in each case is the central problem in nonthermal low-temperature plasmas. One approach is offered by solving the Boltzmann equation adapted to plasmas:

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \vec{c}\,\frac{\partial F}{\partial \vec{r}} + \frac{\vec{F}_L}{m_e}\,\frac{\partial F}{\partial \vec{c}} = C(F) \tag{1.1}$$

where $\vec{r}$ is the position vector of the particle and $\vec{c}$ its velocity, and

$$\vec{F}_L = e_o\,(\vec{E} + \vec{c} \times \vec{B}) \tag{1.2}$$

is the Lorentz force. The treatment of this equation has shown considerable progress during the last third of the 20th century, especially with the handling of the complexity of the collision integral $C(F)$ and the time and space variable terms (see, e.g., Chapter 2). However, the problem has not been mastered at all and will continue into the 21st century. Sometimes the approximation of distribution functions by simple formulas is desirable. On this occasion the following standard terms for the electron energy distribution have proved to be valuable:

$$\hat{f}_0\,(U) = \frac{a}{U_e^{3/2}}\,\exp\left[-\frac{1}{m}\,\left(\frac{U}{U_e}\right)^m\right] \quad \text{with} \quad m > 0 \tag{1.3}$$

and where

$$a = (1/m)^{(3-2m)/2m}/\Gamma(3/2m) \tag{1.4}$$

(see Table 1.2). A possible dependence on space and time of the kinetic energy $e_0 U$ may be incorporated in the distribution parameter $U_e = U_e(\vec{r}, t)$. With $m = 1$, Eq. (1.3) yields the Maxwell distribution, while $m = 2$ represents the Druyvesteyn distribution. Of course the application of standard energy distributions in plasma physics is restricted to special cases, e.g., if only electrons with kinetic energies near the mean energy are of importance.

Table 1.2 Values of a for given m.

m	1	2	3
a	1.128	0.970	0.798

1.4.2
Kinetic Temperature of Electrons

The kinetic temperature T_e of an electron gas is defined by means of the mean energy U_{em},

$$\frac{3}{2}\,kT_e = e_0\,U_{em} \quad \text{with} \quad U_{em} = \int_0^\infty U^{3/2}\,\hat{f}_0\,(U)\,dU \tag{1.5}$$

This results in

$$T_e\ (\text{kelvin}) \mathrel{\hat{=}} 7734\, U_{em}\ (\text{volt})$$

Using Eq. (1.3), we obtain

$$U_{em} = \varepsilon\, U_e \quad \text{with} \quad \varepsilon \; = \; m^{1/m}\Gamma(5/2m)/\Gamma(3/2m) \tag{1.6}$$

(see Table 1.3). For the Maxwell distribution we thus obtain

$$T_e\ (\text{kelvin}) \mathrel{\hat{=}} 11,600\, U_e\ (\text{volt})$$

In most cases the approximation 1 V $\simeq\ 10^4$ K is sufficient.

Table 1.3 Values of ε for given m.

m	1	2	3
ε	1.5	1.046	0.856

1.4.3
Coefficients for Particle and Energy Transport

The electron flow density depends on the electron density n_e and the drift velocity $\vec{v}_e$ and is given by

$$\vec{j_e} \; = \; n_e\, \vec{v}_e \; = \; -\; n_e\, \mu_e\, \vec{E} \; - \; \text{grad}\,(n_e D_e) \tag{1.7}$$

The first term on the right-hand side represents the electrical field drift, the second one combines the action of diffusion and thermodiffusion. Using the usual approximations the mobility μ_e and the diffusion coefficient D_e are given by

$$\mu_e \; = \; -\,\frac{1}{3}\left(\frac{2e_0}{m_e}\right)^{1/2} \int_0^\infty \lambda_e U\, \frac{\partial \hat{f}_0}{\partial U}\, dU \tag{1.8}$$

$$D_e \; = \; \frac{1}{3}\left(\frac{2e_0}{m_e}\right)^{1/2} \int_0^\infty \lambda_e\, U\, \hat{f}_0\, dU \tag{1.9}$$

The particle flow is connected with a flow of energy,

$$\vec{j_e^*} \; = \; n_e U_{em}\, \vec{v}_e^* \; = \; -\, n_e\, \mu_e^* U_{em} \vec{E} \; - \; \text{grad}\,(n_e\, U_{em} D_e^*) \tag{1.10}$$

$$U_{em}\, \mu_e^* \; = \; -\;\frac{1}{3}\left(\frac{2e_0}{m_e}\right)^{1/2} \int_0^\infty \lambda_e U^2\, \frac{\partial \hat{f}_0}{\partial U}\, dU \tag{1.11}$$

$$U_{em} D_e^* \; = \; \frac{1}{3}\left(\frac{2e_0}{m_e}\right)^{1/2} \int_0^\infty \lambda_e U^2\, \hat{f}_0\, dU \tag{1.12}$$

Using a simple exponential approximation for the energy dependence of the free path λ_e for momentum transfer of electrons:

$$\frac{1}{\lambda_e N} = \sigma_m(U) = a_m U^n \tag{1.13}$$

where N is the particle density of the neutral gas and $\sigma_m(U)$ is the cross section for momentum transfer; the standard distribution equation (1.3) results in the following transport coefficients:

$$\begin{aligned} \mu_e &= \frac{\mu}{a_m N}\left(\frac{2e_0}{m_e}\right)^{1/2} U_e^{-(2n+1)/2} \\ D_e &= \alpha U_e \mu_e \qquad \text{Nernst–Townsend–Einstein relation} \\ D_e^* &= \alpha^* U_e \mu_e \\ \mu_e^* &= \gamma \mu_e \end{aligned} \tag{1.14}$$

with

$$\mu = \frac{m^{\frac{2m-2n-1}{2m}}}{3} \frac{\Gamma\left(\frac{1+m-n}{m}\right)}{\Gamma\left(\frac{3}{2m}\right)} \quad \alpha = m^{\frac{1-m}{m}} \frac{\Gamma\left(\frac{2-n}{m}\right)}{\Gamma\left(\frac{1+m-n}{m}\right)} \tag{1.15}$$

$$\alpha^* = m^{\frac{1-m}{m}} \frac{\Gamma\left(\frac{3}{2m}\right)}{\Gamma\left(\frac{5}{2m}\right)} \frac{\Gamma\left(\frac{3-n}{m}\right)}{\Gamma\left(\frac{1+m-n}{m}\right)} \quad \gamma = \frac{\Gamma\left(\frac{3}{2m}\right)}{\Gamma\left(\frac{5}{2m}\right)} \frac{\Gamma\left(\frac{2+m-n}{m}\right)}{\Gamma\left(\frac{1+m-n}{m}\right)} \tag{1.16}$$

1.4.4
Generalized Boltzmann Equilibrium

In front of insulating walls or floating metallic surfaces a plasma shows, as a matter of principle, inhomogeneities which are similar to the Boltzmann equilibrium in a neutral gas under the action of external forces. These inhomogeneities should not be confused with sheath regions because there is no violation of the plasma conditions. In particular, the quasineutrality remains in force. Of course important differences from the Boltzmann equilibrium with neutral gases exist in the plasma:

- The origin of the forces is space charges in the plasma.
- Deviations from the Maxwell distribution function must be taken into consideration.
- The distribution function may vary in space.

The first condition for the Boltzmann equilibrium is a vanishing particle flow in the flow direction. Then Eq. (1.7) yields

$$\vec{E} = -\frac{1}{\mu_e} \frac{\text{grad}\,(n_e D_e)}{n_e} \tag{1.17}$$

In the case of a Maxwell distribution which is constant in space, we have

$$\vec{E} = -U_e \frac{\text{grad}\, n_e}{n_e} = -\text{grad}\, V \tag{1.18}$$

and the concentration $n_e(\vec{r})$ shows the well-known exponential behavior under the action of the potential $V(\vec{r})$ according to the barometric formula:

$$\frac{n_e\,(\vec{r})}{n_e(0)} \;=\; \exp\left(\frac{V(\vec{r})}{U_e}\right) \tag{1.19}$$

In this case the properties of the Maxwell distribution also provide for the energetic equilibrium of the electrons ($\vec{j}_e^{\,*} = 0$). Every generalization (e.g., $U_e = U_e(\vec{r})$) has to ensure additionally that

$$\vec{j}_e^{\,*} = 0\,, \quad \text{resulting in} \quad \vec{E} \;=\; -\frac{1}{U_{em}\,\mu_e^*}\,\frac{\text{grad}\,(n_e\,U_{em}\,D_e^*)}{n_e} \tag{1.20}$$

Equations (1.17) and (1.20) determine the necessary conditions of the spatial variables $U_e(\vec{r})$, $V(\vec{r})$, $n_e(\vec{r})$ for generalized equilibrium. Hence for the standard distributions (Eq. (1.3)) and with Eq. (1.13) it follows that

$$\frac{\text{grad}\,U_e}{U_e} = \frac{\delta\;\text{grad}\,n_e}{n_e} \qquad \delta = \frac{\alpha^* - \alpha\gamma}{(n-1/2)(\alpha^* - \alpha\gamma) - \alpha^*} \tag{1.21}$$

$$\vec{E} = -\alpha\left(1 + \left(\frac{1}{2} - n\right)\delta\right)\frac{U_e\;\text{grad}\,n_e}{n_e} \qquad \frac{U_e(\vec{r})}{U_e(0)} = \left(\frac{n_e(\vec{r})}{n_e(0)}\right)^{\delta} \tag{1.22}$$

For the Maxwell distribution, which holds for $U_e = \text{const.}$, we have $\delta = 0$. For non-Maxwell distributions ($\delta \neq 0$) we obtain

$$\frac{n_e(\vec{r})}{n_e(0)} \;=\; \left(\frac{\delta}{\alpha(1+(1/2-n)\delta)}\,\frac{V(\vec{r})}{U_e(0)} + 1\right)^{1/\delta} \tag{1.23}$$

Consideration of the spatial variations of the electron distribution function is of utmost importance in the case of deviations from the Maxwell distribution. Such deviations are common in plasmas, mostly as a consequence of collisions with heavy particles. Consequently a detailed analysis of the energetic relations of the electron gas is necessary. Very recently the nonlocal complex nature of the power and momentum balance in space-dependent plasmas was studied for the first time, starting from the Boltzmann equation (Winkler 1996, see also Chapter 2). Neglecting collisions, the predominance and stability of the Maxwell distribution (e.g., in low-pressure discharges, Langmuir paradox) should be a consequence of an energetic quasiequilibrium $\vec{j}_e^{\,*} \approx 0$.

Table 1.4 shows numerical values for the particle and energy transport, using standard distributions, while Figs. 1.1 and 1.2 contain some illustrations of the Boltzmann equilibrium and its generalization.

1.4.5
Ambipolar Diffusion

Within the plasma the movements of ions and electrons are interconnected via electric space charges. In the absence of external forces these space charges provide for equal electron and

Table 1.4 Numerical values for particle and energy transport, calculated for three different standard distributions.

	$m = 1$ (Maxwell distribution)					
	$n = -1$	–1/2	0	1/6	1	3 /2
α	1	1	1	1	1	1
α^*	2	1.667	1.333	1.222	0.667	0.333
γ	2	1.667	1.333	1.222	0.667	0.333
δ	0	0	0	0	0	0
μ	0.752	0.5	0.376	0.354	0.376	0.667
	$m = 2$ (Druyvesteyn distribution)					
	$n = -1$	–1/2	0	1/6	1	3/2
α	0.627	0.697	0.798	0.842	1.253	2.093
α^*	0.956	0.956	0.956	0.956	0.956	0.95
γ	1.797	1.667	1.524	1.476	1.198	1
δ	0.243	0.275	0.315	0.333	0.444	0.543
μ	0.647	0.5	0.406	0.383	0.324	0.333
	$m = 4$					
	$n = -1$	–1/2	0	1/6	1	3/2
α	0.489	0.571	0.692	0.746	1.282	2.443
α^*	0.659	0.716	0.789	0.820	1.036	1.269
γ	1.713	1.667	1.616	1.579	1.498	1.427
δ	0.453	0.488	0.522	0.534	0.598	0.636
μ	0.593	0.5	0.429	0.409	0.334	0.307

ion drifts. For instance in the direction toward isolating walls the steady-state drift velocities of electrons and ions converge to the common velocity v_{am} of ambipolar diffusion ($n_e \approx n_i = n$)

$$\begin{aligned} v_{\text{am}} &= -\,D_{\text{am}}\,\frac{\text{grad}\,n}{n} \\ \text{where}\quad D_{\text{am}} &= \frac{\mu_e\,D_i\;+\;\mu_i\,D_e}{\mu_e\;+\;\mu_i} \end{aligned} \tag{1.24}$$

is defined as the *ambipolar diffusion coefficient.* Generally we have $\mu_e \gg \mu_i$, which results in

$$D_{\text{am}} \;\approx\; \alpha\,\mu_i\,U_e \tag{1.25}$$

The regime of ambipolar diffusion ($j_e = j_i$) shows some analogy to the Boltzmann equilibrium ($j_e = j_i = 0$). Therefore the formulas of Section 1.4.4 are also valid approximately for ambipolar diffusion. Instead of formula (1.17) we obtain for the internal electric ambipolar field:

$$\vec{E}_{\text{am}} = -\frac{1}{\mu_e}\left(1 - \frac{\mu_i}{\mu_e}\right)\frac{\text{grad}\,(n_e D_e)}{n_e} \tag{1.26}$$

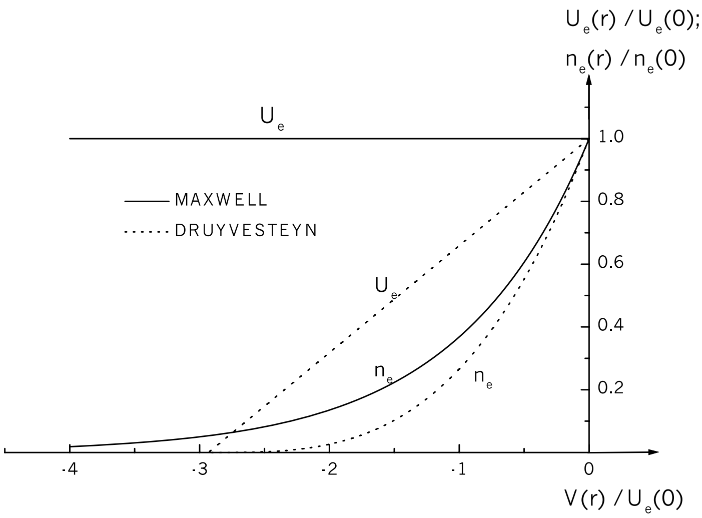

Fig. 1.1 Equilibrium values of the electron concentration and temperature for given electrical potentials (full curves: Boltzmann equilibrium, i.e., Maxwell distribution with $U_e = \mathrm{const.}$; dashed curves: Generalization for a Druyvesteyn distribution with $U_e = U_e(r)$ and $n = 0$ in Eq. (1.13)).

1.4.6 Condition of Quasineutrality

In order to guarantee the status of free particles for electrons and ions in the plasma the field energy of space charges is limited to values much less than the kinetic energy of the charge carriers. This results in tolerable deviations $\Delta n_e = n_{eo} - n_e$ from exact neutrality $n_{eo} = n_{io}$:

$$| \Delta n_e|/n_{eo} \ \leq \lambda_D/L \tag{1.27}$$

where λ_D is the Debye screening length (see Section 1.4.7), and L is a characteristic plasma length (e.g., the radius of a plasma column). Within the Debye length considerable deviations from neutrality may occur in plasmas. In this case the dynamics of such deviations is governed by the Langmuir plasma frequency.

1.4.7 Debye Screening Length

The electrical potential distribution of a charge carrier inside a plasma is different from the corresponding distribution in a vacuum. In a plasma each charge-carrier polarizes its surroundings and thereby reduces the interaction length of the Coulomb potential V_c which is compensated in part by the space charge potential V_R (see Fig. 1.3). In the case of ions of charge Ze_0 and with $e_0 V_D \ll kT_i$ the screened potential is

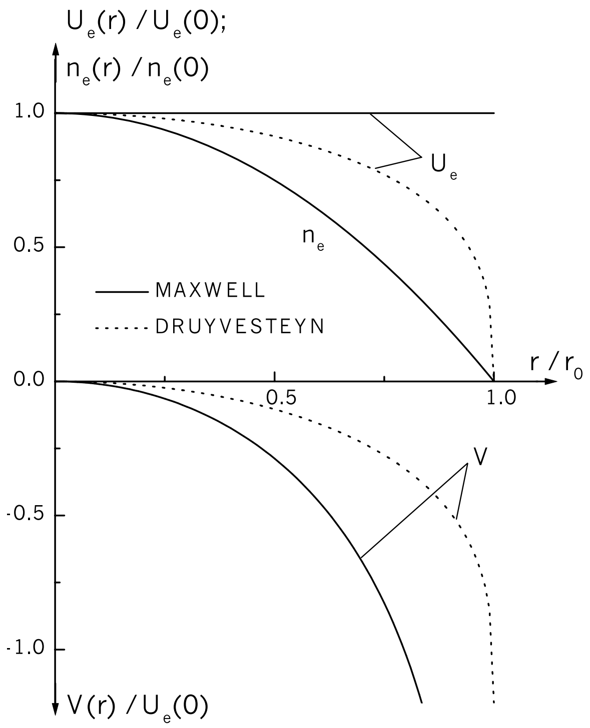

Fig. 1.2 Equilibrium values of the electron temperature and the electrical potential for a preset concentration profile $n_e(r)/n_e(0) = 1 - (r/r_0)^2$; curves according to the conditions of Fig. 1.1.

$$V_D(r) = \frac{1}{4\pi\varepsilon_0}\frac{e_0}{r}\,\exp\left(-\frac{r}{\lambda_D}\right) \qquad \lambda_D^2 = \frac{\varepsilon_0\,kT_e}{e_0^2\,n_{e0}\,(1 + ZT_e/T_i)} \tag{1.28}$$

Outside the Debye length λ_D the potential may be neglected ($V_D \approx 0$). This cutoff is typical for plasma conditions and of great importance for the interaction of charge carriers. Equation (1.28) is based on the assumption that in a sphere of radius λ_D many charge carriers exist, i.e.,

$$\frac{4\pi}{3}\,n_{e0}\,\lambda_D^3 \gg 1 \tag{1.29}$$

Then the plasma state is termed *ideal*. In this case the Coulomb interaction energy between two charge carriers at the mean distance is much smaller than the thermal energy. For *nonideal* plasmas the electrostatic energies exceed the thermal energy. Figure 1.4 displays calculated values of the Debye length.

1.4.8 Degree of Ionization

Calculation of the degree of ionization in a closed form is only possible for plasmas in the exact equilibrium state. The Saha–Eggert equation then holds. For single ionization the following equation is valid:

$$\frac{x^2}{1-x^2} = 2\left(\frac{2\pi m_e}{h^3}\right)^{3/2} \frac{g_1 E_0^{5/2}}{g_0 p} \left(\frac{kT}{E_0}\right)^{5/2} \exp\left(-\frac{E_0}{kT}\right) \tag{1.30}$$

where $x = n_e/(n_0 + n_e)$ is the degree of ionization, $p = (n_0 + 2n_e)kT$ is the kinetic pressure, g_0 and g_1 are statistical weights, and E_0 is the ionization energy.

Figure 1.5 shows according to Eq. (1.30) some curves of constant degree of ionization. Under the nonequilibrium conditions of LTP the calculation of n_e or x requires a detailed analysis of the corresponding balances. The energy balance of the electrons then results in a very simple and useful expression for the estimation of the degree of ionization. In the steady state and for

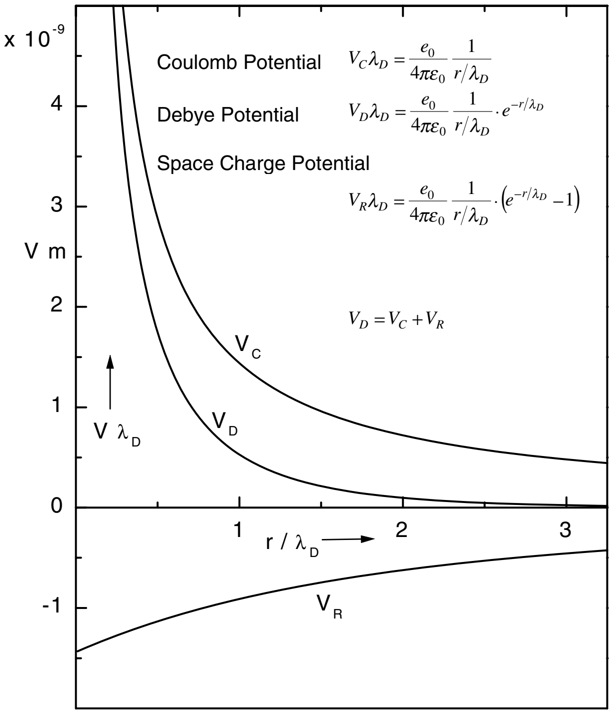

Fig. 1.3 Variation of the electrical potential around an ion imbedded in a plasma.

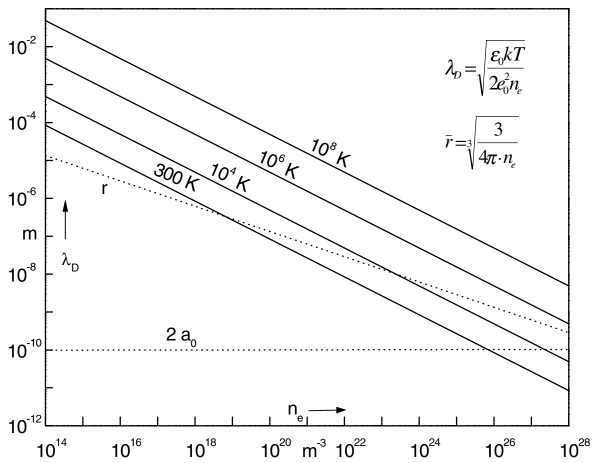

Fig. 1.4 Debye screening length λ_D versus electron concentration ($\bar{r}$, mean distance of electrons; a_0, radius of the first Bohr orbit).

$n_e \ll n$ it reads

$$\frac{n_e}{n} \approx \frac{2}{3}\,\frac{1}{\delta_{\text{loss}}}\,\frac{\tau_e\, n}{e_0\, U_e}\,\frac{P/V}{n^2} \tag{1.31}$$

where τ_e is the mean free time between electron collisions, P/V is the power density supplied to the plasma and δ_{loss} is the mean fraction of energy that the electrons lose in a single collision. For elastic collisions, $\delta_{\text{loss}} = \delta_{\text{el}} = 2m_e/M$ is of the order of $\delta_{\text{el}} \approx 10^{-4}$–$10^{-5}$. The inelastic energy loss δ_{inel} is typically larger by one to two orders of magnitude. The quantity $\tau_e n/\delta_{\text{loss}}$ is a function of U_e, but often its variations are rather small. Compared to the range of P/Vn^2 it may be regarded as constant. This is a rather good approximation at higher gas pressures ($p \geq 10^3$ Pa).

1.4.9
Electrical Conductivity

Under the action of an electrical field $\vec{E}$ the free electrons and ions of the plasma gain drift velocities and generate an electric current density of

$$\vec{j} \;=\; -\; e_0\,(n_e\,\vec{\nu}_e - n_i\,\vec{\nu}_i) \;=\; e_0\,(n_e\,\mu_e + n_i\,\mu_i)\,\vec{E} \tag{1.32}$$

Generally, since of $\mu_e \gg \mu_i$ and $n_e \approx n_i$, only the contribution of electrons determines the current density. Then the electrical conductivity σ of a plasma is given by

$$\sigma \;=\; e_0\, n_e\, \mu_e \tag{1.33}$$

The concentration n_e and mobility μ_e of the electrons are given by, e.g., Eqs. (1.8) and (1.31). The elementary kinetic approximation for the conductivity reduces to

$$\sigma \;=\; e_0^2 n_e \tau_e / m_e \tag{1.34}$$

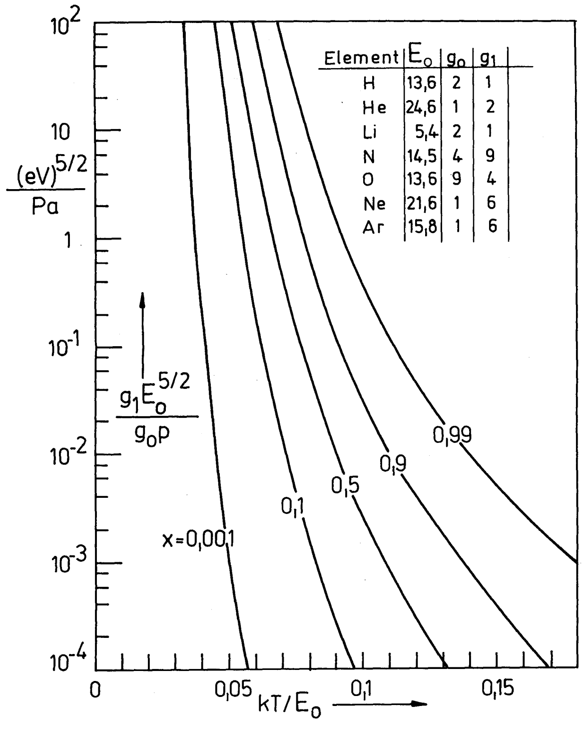

Fig. 1.5 Curves for constant degree of ionization in conformity with the Saha–Eggert equation.

With regard to the degree of ionization two cases may be distinguished:

- *Weakly ionized plasmas.* The mean free time of flight τ_e is defined by electron–atom collisions and is independent of n_e. Consequently, $\sigma \propto n_e$.

- *Fully ionized plasmas.* τ_e is determined by Coulomb collisions with $\tau_e \propto 1/n_e$ and the conductivity is constant.

For a fully ionized thermal plasma the corresponding equation is named the Spitzer formula

$$\sigma = \frac{64\sqrt{2\pi}\,\varepsilon_0^2}{e_0^2\,\sqrt{m_e}} \frac{(kT_e)^{3/2}}{\ln\,\Lambda} \tag{1.35}$$

where $\ln \Lambda$ is the Coulomb logarithm; its numerical value for the majority of plasmas lies in the range from 15 to 20.

1.4.10
Plasma Frequency

Representative of the multitude of dynamic processes in plasmas is the longitudinal electrical oscillations (Langmuir 1928). The occurrence of space charges generates in general a quasielastic coupling of the electrons to the ionic background and results in oscillations with a frequency ω_P given by

$$\omega_P^2 = \frac{e_0^2 \, n_e}{\varepsilon_0 \, m_e} \tag{1.36}$$

The electron plasma frequency (Eq. (1.36)) is critical for the propagation of electromagnetic waves in plasmas. In the range $\omega < \omega_P$ the damping of the waves is strong. For $\omega = \omega_P$ electromagnetic waves show strong reflection at the plasma interface. This is related to the refractive index $n^2 = 1 - (\omega_P/\omega)^2$ (Eccle relation).