

## TABLE OF CONTENTS

PART I BASIC CONCEPTS

Chapter 1	Introduction	1
1.1	Inverse problems	1
1.2	Some examples of inverse problems	4
1.3	Analysis of inverse problems	14
Chapter 2	Ill-posed problems	16
2.1	General properties	16
2.2	Restoration of continuity in the linear case	18
2.3	Stability estimates	23
Chapter 3	Regularization	27
3.1	Reconstruction from non-exact data	27
3.2	Preliminary results on Tikhonov's method	34
3.3	Regularizing schemes	37
3.4	A tutorial example: The reconstruction of a derivative	41
3.5	Optimal reconstruction of linear functionals	43

PART II REGULARIZATION METHODS

Chapter 4	The singular value decomposition	49
4.1	Compact operators	49
4.2	The spectrum of compact selfadjoint operators	53
4.3	The singular value decomposition	57
4.4	The min-max principle	64
4.5	The asymptotics of singular values	66
4.6	Picard's criterion	70
Chapter 5	Applications of the singular value decomposition	73
5.1	Hilbert scales	73
5.2	Convergence of regularizing schemes	78
5.3	On the use of the conjugate gradient method	82
5.4	n-widths	88

Chapter 6	The method of Tikhonov	91
6.1	The generalized inverse	91
6.2	The classical method of Tikhonov	97
6.3	Error bounds for Tikhonov regularization in Hilbert scales	99
6.4	On discrepancy principles	102
6.5	Discretization in Tikhonov's method	109
Chapter 7	Regularization by discretization	112
7.1	Discretization by projection methods	112
7.2	Quasioptimality and robustness	116
7.3	Specific methods	120
7.4	Asymptotic estimates	123
<u>PART III</u>	<u>LEAST SQUARES SOLUTIONS OF SYSTEMS OF LINEAR EQUATIONS</u>	
Chapter 8	Least squares problems	127
8.1	The singular value decomposition of a matrix	127
8.2	The pseudo-inverse	133
8.3	Least squares solutions	138
8.4	Perturbation results	141
8.5	Application: Fitting of data	149
Chapter 9	Numerical aspects of least squares problems	152
9.1	Calculation of $A^+$ : The factorization approach	152
9.2	Rank decision	158
9.3	Cross-validation	161
9.4	Successive approximation	164
9.5	The ART-algorithm	171
<u>PART IV</u>	<u>SPECIFIC TOPICS</u>	
Chapter 10	Convolution equations	179
10.1	The Fourier transform	179
10.2	Regularization of convolution equations	183
10.3	On the discretization of convolution equations	190
10.4	Reconstruction by successive approximation	195

Chapter 11 The final value problem	203
11.1 Introduction	203
11.2 The mild solution of the forward problem	207
11.3 The Hilbert scales $E_{a,t}$	211
11.4 Regularizing schemes	214
Chapter 12 Parameter identification	221
12.1 Identifiability of parameters in dynamical systems	221
12.2 Identification in linear dynamic systems	224
12.3 Identification in bilinear structures	228
12.4 Adaptive identification	237
REFERENCES	245
NOTATIONS	251
SUBJECT INDEX	252