CONTENTS

CHAPTER I : DIFFERENTIAL MANIFOLDS

Α.	FROM SUBMANIFOLDS TO ABSTRACT MANIFOLDS	
	Submanifolds of \mathbb{R}^{n+k}	1
	Abstract manifolds	5
	Smooth maps	10
в.	TANGENT BUNDLE	
	Tangent space to a submanifold of \mathbb{R}^{n+k}	12
	The manifold of tangent vectors	13
	Vector bundles	15
	Differential map	16
C.	VECTOR FIELDS:	
	Definitions	17
	Another definition for the tangent space	18
	Integral curves and flow of a vector field	22
	Image of a vector field under a diffeomorphism	23
D.	BABY LIE GROUPS	
	Definitions	26
	Adjoint representation	28
$\mathbf{E}.$	COVERING MAPS AND FIBRATIONS	
	Covering maps and quotient by a discrete group	29
	Submersions and fibrations	30
	Homogeneous spaces	31
F.	TENSORS	
	Tensor product (digest)	34
	Tensor bundles	35
	Operations on tensors	36
	Lie derivatives	37
	Local operators, differential operators	38 39
	A characterization for tensors	39
$\mathbf{G}.$	EXTERIOR FORMS	
	Definitions	40
	Exterior derivative	41
	Volume forms	44
	Integration on an oriented manifold	45 46
	Haar measure on a Lie group	40

CHAPTER II: RIEMANNIAN METRICS

A .	EXISTENCE THEOREMS AND FIRST EXAMPLES	
	Definitions	49
	First examples	51
	Examples: Riemannian submanifolds, product Riemannian manifolds	55
	Riemannian covering maps, flat tori	56
	Riemannian submersions, complex projective space	61
	Homogeneous Riemannian spaces	63
В.	COVARIANT DERIVATIVE	
	Connexions	67
	Canonical connexion of a Riemannian submanifold	69
	Extension of the covariant derivative to tensors	70
	Covariant derivative along a curve	72
	Parallel transport	75
	Examples	75
C.	GEODESICS	
	Definitions	77
	Local existence and uniqueness for geodesics, exponential map	81
	Riemannian manifolds as metric spaces	84
	Complete Riemannian manifolds, Hopf-Rinow's theorem	90
	Geodesics and submersions, geodesics of P^n C	94
	Cut locus	96
СНА	PTER III : CURVATURE	
A.	THE CURVATURE TENSOR	
	Second covariant derivative	102
	Algebraic properties of the curvature tensor	103
	Computation of curvature : some examples	105
	Ricci curvature, scalar curvature	107

B. FIRST SECOND VARIATION OF ARC-LENGTH AND ENER	.GY
Technical preliminaries: vector fields along parameterized	
submanifolds	108
First variation formula	110
Second variation formula	111
C. JACOBI VECTOR FIELDS	
Basic topics about second derivatives	113
Index form	114
Jacobi fields and exponential map	116
Applications: S^n , H^n , P^n R, 2-dimensional manifolds	118
D. RIEMANNIAN SUBMERSIONS AND CURVATURE	
Riemannian submersions and connexions	120
Jacobi fields of $P^n\mathbf{C}$	121
O'Neill's formula	123
Curvature and length of small circles. Application to	
Riemannian submersions	124
E. THE BEHAVIOR OF LENGTH AND ENERGY IN THE NEIGHBORHOOD OF A GEODESIC	
The Gauss lemma	126
Conjugate points	127
Some properties of the cut-locus	130
F. MANIFOLDS WITH CONSTANT SECTIONAL CURVATURE	E
Spheres, Euclidean and hyperbolic spaces	131
G. TOPOLOGY AND CURVATURE	
The Myers and Cartan theorems	133
H. CURVATURE AND VOLUME	
Densities on a differential manifold	135
Canonical measure of a Riemannian manifold	136
Examples: spheres, hyperbolic spaces, complex projective spaces	138
Small balls and scalar curvature	139
Volume estimates	140
I. CURVATURE AND GROWTH OF THE FUNDAMENTAL GRO	OUP
Growth of finite type groups	144
Growth of the fundamental group of compact manifolds with negative	
curvature	145

J. CURVATURE AND TOPOLOGY	
Introduction	147
Traditional point of view: pinched manifolds	147
Almost flat pinching	148
Coarse point of view: compactness theorems of Gromov and Cheeger	149
K. CURVATURE AND REPRESENTATIONS OF THE ORTHOGONAL GROUP	
Decomposition of the space of curvature tensors	150
Conformally flat manifolds	153
The second Bianchi identity	154
CHAPITRE IV: ANALYSIS ON MANIFOLDS AND THE RICCI CURVATURE	
A. MANIFOLDS WITH BOUNDARY	
Definition	155
The Stokes theorem and integration by parts	156
B. BISHOP'S INEQUALITY REVISITED	
Some commutations formulas	159
Laplacian of the distance function	160
Another proof of Bishop's inequality	161
The Heintze-Karcher inequality	162
C. DIFFERENTIAL FORMS AND COHOMOLOGY	
The de Rham complex	164
Differential operators and their formal adjoints	165
The Hodge-de Rham theorem	167
A second visit to the Bochner method	168
D. BASIC SPECTRAL GEOMETRY	
The Laplace operator and the wave equation	170
Statement of the basic results on the spectrum	172
E. SOME EXAMPLES OF SPECTRA	
Introduction	172
The spectrum of flat tori	174
Spectrum of (S^n, can)	175
F. THE MINIMAX PRINCIPLE	
The basic statements	177

G. THE RICCI CURVATURE AND EIGENVALUES ESTIM	ATES
Introduction	181
Bishop's inequality and coarse estimates	181
Some consequences of Bishop's theorem	182
Lower bounds for the first eigenvalue	184
CHAPTER V: RIEMANNIAN SUBMANIFOLDS	
A. CURVATURE OF SUBMANIFOLDS	
Introduction	185
Second fundamental form	185
Curvature of hypersurfaces	187
Application to explicit computations of curvature	189
B. CURVATURE AND CONVEXITY	
The Hadamard theorem	192
C. MINIMAL SURFACES	
First results	196
SOME EXTRA PROBLEMS	201
SOLUTIONS OF EXERCISES	
Chapter I	203
Chapter II	213
Chapter III	230
Chapter IV	235
Chapter V	236
BIBLIOGRAPHY	241
INDEX	245