

# Contents

Introduction

Notations

Part I

|           |                                                             |    |
|-----------|-------------------------------------------------------------|----|
| Chapter 1 | Hypergeometric Differential Equations .....                 | 1  |
| § 1.1     | Hypergeometric Series .....                                 | 1  |
| § 1.2     | Hypergeometric Equations .....                              | 2  |
| § 1.3     | Contiguity Relations .....                                  | 3  |
| § 1.4     | Euler's Integral Representation .....                       | 5  |
| § 1.5     | Barnes' Integral Representation .....                       | 11 |
| § 1.6     | Confluent Hypergeometric Equations .....                    | 12 |
| Chapter 2 | General Theory of Differential Equations I ....             | 14 |
| § 2.1     | How to Write Differential Equations .....                   | 14 |
| § 2.2     | Cauchy's Fundamental Theorem .....                          | 15 |
| § 2.3     | Monodromy Representations of Differential<br>Equations..... | 16 |
| § 2.4     | Regular Singularities .....                                 | 18 |
| § 2.5     | The Frobenius Method .....                                  | 20 |
| § 2.6     | Fuchsian Equations .....                                    | 24 |
| Chapter 3 | The Riemann and Riemann-Hilbert Problems .....              | 27 |
| § 3.1     | Statement of the Problems .....                             | 27 |
| § 3.2     | An Observation .....                                        | 28 |
| § 3.3     | The Solution of the Riemann Problem .....                   | 29 |
| § 3.4     | Apparent Singularities .....                                | 29 |
| § 3.5     | A Solution of the Riemann-Hilbert Problem .....             | 30 |
| § 3.6     | Isomonodromic Deformations .....                            | 35 |
| Chapter 4 | Schwarzian Derivatives I .....                              | 38 |
| § 4.1     | Definitions and Properties .....                            | 38 |
| § 4.2     | Relations With Differential Equations .....                 | 39 |

|           |                                                                             |     |
|-----------|-----------------------------------------------------------------------------|-----|
| § 4.3     | A Canonical Form of Differential Equations .....                            | 42  |
| § 4.4     | Local Behaviour of Schwarzian Derivative .....                              | 44  |
| Chapter 5 | The Gauss-Schwarz Theory for Hypergeometric<br>Differential Equations ..... | 46  |
| § 5.1     | Orbifolds and Their Uniformizations .....                                   | 46  |
| § 5.2     | Uniformizing Differential Equations .....                                   | 48  |
| § 5.3     | The Gauss-Schwarz Theory .....                                              | 51  |
| Part II   |                                                                             |     |
| Chapter 6 | Hypergeometric Differential Equations in<br>Several Variables .....         | 58  |
| § 6.1     | Hypergeometric Series .....                                                 | 58  |
| § 6.2     | Hypergeometric Differential Equations .....                                 | 60  |
| § 6.3     | Contiguity Relations .....                                                  | 63  |
| § 6.4     | Euler's Integral Representations .....                                      | 65  |
| § 6.5     | The Erdelyi-Takano Integral Representations ....                            | 78  |
| § 6.6     | The Barnes Integral Representations .....                                   | 78  |
| § 6.7     | A Relation Between the Equation $F_D$ and<br>the Garnier System $G_N$ ..... | 79  |
| § 6.8     | Confluent Hypergeometric Equations .....                                    | 81  |
| Chapter 7 | The General Theory of Differential Equations ...                            | 82  |
| § 7.1     | Singularities of Differential Equations .....                               | 82  |
| § 7.2     | Holonomic Systems .....                                                     | 85  |
| § 7.3     | The Equation $F_1$ .....                                                    | 89  |
| § 7.4     | Equation of Rank $n+1$ .....                                                | 90  |
| § 7.5     | Differential Equations on Manifolds .....                                   | 92  |
| § 7.6     | Regular Singularities .....                                                 | 93  |
| Chapter 8 | Schwarzian Derivatives II .....                                             | 95  |
| § 8.1     | Definitions and Properties .....                                            | 95  |
| § 8.2     | Relations With Differential Equations .....                                 | 99  |
| § 8.3     | A Canonical Form of Differential Equations ....                             | 100 |
| § 8.4     | Projective Equations and Projective<br>Connections .....                    | 102 |
| § 8.5     | Local Properties of Schwarzian Derivatives ....                             | 104 |

|                    |                                                                                    |     |
|--------------------|------------------------------------------------------------------------------------|-----|
| Chapter 9          | The Riemann and Riemann-Hilbert Problems II                                        | 107 |
| § 9.1              | The Riemann Problem in Several Variables                                           | 107 |
| § 9.2              | Accessory Parameters                                                               | 110 |
| Chapter 10         | The Gauss-Schwarz Theory in Two Variables                                          | 115 |
| § 10.1             | Uniformizability of Orbifolds                                                      | 116 |
| § 10.2             | Orbifolds Whose Universal Uniformizations<br>Are Symmetric Spaces                  | 118 |
| § 10.3             | Line Arrangements in $\mathbf{P}_2$ and Orbifolds<br>Uniformized by $\mathbf{B}_2$ | 124 |
| § 10.4             | Uniformizing Differential Equations                                                | 135 |
| § 10.5             | The Gauss-Schwarz Theory for Appell's<br>Equation $F_1$                            | 137 |
| § 10.6             | The Monodromy Representation of Appell's<br>Equation $F_1$                         | 145 |
| Chapter 11         | Reflection Groups                                                                  | 154 |
| § 11.1             | Unitary Reflection Groups                                                          | 155 |
| § 11.2             | Unitary Reflection Groups of Dimension 2                                           | 159 |
| § 11.3             | Parabolic Reflection Groups of Dimension 2                                         | 170 |
| § 11.4             | Line Arrangements Defined by Unitary<br>Reflection Groups of Dimensions 3 and 4    | 179 |
| Chapter 12         | Toward Finding New Differential Equations                                          | 181 |
| § 12.1             | Tensor Forms Associated to the Differential<br>Equations                           | 181 |
| § 12.2             | Integrability Conditions                                                           | 185 |
| § 12.3             | The Restricted Riemann Problem                                                     | 188 |
| § 12.4             | G-Invariant Differential Equations I                                               | 191 |
| § 12.5             | G-Invariant Differential Equations II                                              | 198 |
| References         |                                                                                    | 204 |
| Sources of Figures |                                                                                    | 215 |