# Contents

**Continuum Thermodynamic and Rate Variational Formulation of Models for Extended Continua** ..... 1
*Bob Svendsen*
1 Introduction ..... 1
2 Energy Balance and Basic Constitutive Assumptions ..... 3
3 Euclidean Frame-Indifference of the Energy Balance ..... 5
4 Material Frame-Indifference of the Free Energy Density ..... 7
5 Dissipation Principle and Reduced Evolution-Field Relations ..... 8
6 Variational Formulation ..... 10
7 Discussion ..... 13
References ..... 15

**From Lattice Models to Extended Continua** ..... 19
*Stefan Diebels, Daniel Scharding*
1 Introduction ..... 19
2 Lattice Models ..... 21
2.1 Honeycomb Structure ..... 21
2.2 Effective Shear Modulus ..... 23
2.3 Reference Data ..... 25
3 Extended Continuum Theories ..... 26
3.1 The Linear Cosserat Theory ..... 27
3.2 Analytical Solution for Shear ..... 28
4 Parameter Identification ..... 30
4.1 Gradient-Based Methods ..... 30
4.2 Evolution Strategies ..... 32
5 Conclusions and Outlook ..... 34
References ..... 35
Appendix ..... 38

Bibliografische Informationen
http://d-nb.info/1012996271

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

**Rotational Degrees of Freedom in Modeling Materials with Intrinsic Length Scale** ..... 47
*Elena Pasternak, Hans-Bernd Mühlhaus, Arcady V. Dyskin*
1 Introduction ..... 47
2 Non-standard Continua for Modeling Materials with Microstructure ..... 48
3 Homogenization of 1D Structures ..... 52
3.1 Homogenization by Differential Expansion ..... 53
3.2 Homogenization by Integral Transformation (Non-local Cosserat Continuum) ..... 54
3.3 Harmonic Waves in 1D Structures ..... 57
4 Homogenization by Differential Expansions in 3D ..... 58
5 Cosserat Model of Layered Materials with Sliding Layers and Stress Concentrations ..... 59
6 Path-Independent Integrals in Cosserat Continuum ..... 61
7 Conclusions ..... 64
References ..... 65

**Micromorphic vs. Phase-Field Approaches for Gradient Viscoplasticity and Phase Transformations** ..... 69
*Samuel Forest, Kais Ammar, Benoît Appolaire*
1 Introduction ..... 69
2 Thermomechanics with Additional Degrees of Freedom ..... 71
2.1 General Setting ..... 71
2.2 Micromorphic Model as a Special Case ..... 73
2.3 Phase-Field Model as a Special Case ..... 74
3 Constitutive Framework for Gradient and Micromorphic Viscoplasticity ..... 75
3.1 Introduction of Viscous Generalized Stresses ..... 75
3.2 Decomposition of the Generalized Strain Measures ..... 77
4 Phase-Field Models for Elastoviscoplastic Materials ..... 78
4.1 Coupling with Diffusion ..... 80
4.2 Partition of Free Energy and Dissipation Potential ..... 81
4.3 Multi-phase Approach for the Mechanical Contribution ..... 83
4.4 Voigt/Taylor Model Coupled Phase-Field Mechanical Theory ..... 85
5 Conclusion ..... 86
References ..... 86

**Geometrically Nonlinear Continuum Thermomechanics Coupled to Diffusion: A Framework for Case II Diffusion** ..... 89
*Andrew T. McBride, Swantje Bargmann, Paul Steinmann*
1 Introduction ..... 89
2 Preliminaries: Notation and Key Concepts ..... 91

3 Governing Equations ..... 94
3.1 Conservation of Solid Mass ..... 94
3.2 Conservation of Diffusing Species Mass ..... 94
3.3 Balance of Linear and Angular Momentum ..... 96
3.4 Balance of Internal Energy ..... 96
3.5 Balance of Entropy ..... 97
3.6 Constitutive Relations ..... 99
3.7 Temperature Evolution Equation ..... 101
4 Key Features of the Helmholtz Energy Required to Reproduce Case II Diffusion ..... 102
4.1 Energy Associated with Viscoelastic Effects ..... 103
4.2 Energy Associated with Mixing ..... 104
5 Discussion and Conclusions ..... 104
References ..... 106

**Effective Electromechanical Properties of Heterogeneous Piezoelectrics** ..... 109
*Marc-André Keip, Jörg Schröder*
1 Introduction ..... 109
2 Boundary Value Problems on the Macro- and the Mesoscale ..... 112
2.1 Macroscopic Electro-Mechanically Coupled BVP ..... 112
2.2 Mesoscopic Electro-Mechanically Coupled BVP ..... 114
3 Effective Properties of Piezoelectric Materials ..... 116
4 Numerical Example ..... 120
4.1 Invariant Formulation and Material Parameters ..... 121
4.2 Investigation of the "Wolfgang Ehlers 60" Mesostructure ..... 122
5 Conclusion ..... 124
References ..... 125

**Coupled Thermo- and Electrodynamics of Multiphasic Continua** ..... 129
*Bernd Markert*
1 Mixture and Porous Media Theories ..... 129
1.1 The Macroscopic Mixture Approach ..... 130
1.2 Volume Fractions, Saturation and Density ..... 130
2 Kinematical Relations ..... 132
2.1 Mixture Kinematics ..... 132
2.2 Deformation and Strain Measures ..... 134
3 Some Aspects of Electrodynamics ..... 138
3.1 Preliminaries ..... 138
3.2 The Macroscopic Maxwell Equations ..... 139
3.3 Fusion of Electrodynamics and Thermodynamics ..... 141

4 Balance Relations ..... 142
4.1 Stress Concept and Dual Variables ..... 142
4.2 Master Balance Principle for Mixtures ..... 144
5 Conclusion ..... 150
References ..... 151

**Ice Formation in Porous Media** ..... 153
*Joachim Bluhm, Tim Ricken, Moritz Bloßfeld*
1 Introduction ..... 153
2 Basics ..... 155
3 Simplified Quadruple Model ..... 157
3.1 Field Equations ..... 157
3.2 Constitutive Theory ..... 158
4 Examples ..... 166
4.1 Example 1: Capillary Suction during Freezing ..... 167
4.2 Example 2: Heat of Fusion during Phase Transition ..... 170
5 Conclusion ..... 171
References ..... 172

**Optical Measurements for a Cold-Box Sand and Aspects of Direct and Inverse Problems for Micropolar Elasto-Plasticity** ..... 175
*Rolf Mahnken, Ismail Caylak*
1 Introduction ..... 175
2 Specimens and Testing Equipment ..... 178
2.1 Materials and Specimen Preparation ..... 178
2.2 Experimental Equipment ..... 178
3 Uniaxial Compression and Tension Tests ..... 180
3.1 SD-Effect and Optical Measurements ..... 180
3.2 Rate Dependency and Reproducibility ..... 183
3.3 Influence of Storage Time ..... 183
4 Thermo-Mechanical Characterization ..... 184
4.1 Heat Exchanger Variation for Thermal Loading ..... 184
4.2 Mechanical Loading for Different Isothermal Conditions ..... 186
5 Triaxial Characterization ..... 187
6 Modeling of Micropolar Continua ..... 188
6.1 Basic Equations ..... 188
6.2 Yield Function and Plastic Potential ..... 189
7 Direct and Inverse Problems for Micropolar Solids ..... 191
7.1 Direct Problem: Weak Formulation ..... 191
7.2 Inverse Problem: Constrained Least Squares Problem ..... 192
8 Summary and Outlook ..... 194
References ..... 195

**Model Reduction for Complex Continua – At the Example of Modeling Soft Tissue in the Nasal Area** ..... 197
*Annika Radermacher, Stefanie Reese*
1 Indroduction ..... 197
2 Model Reduction for Non-linear Structural Mechanics ..... 200
2.1 SVD-Based Reduction ..... 200
2.2 Error Definition ..... 203
3 Biomechanical Structural Applications ..... 203
3.1 Examples ..... 203
3.2 Study of Convergence ..... 205
3.3 Study of Parameters ..... 205
3.4 Human Nose Model ..... 213
4 Conclusion ..... 215
References ..... 215

**Author Index** ..... 219