## **CONTENTS**

	duction: Population Without Age	1
HILLO	oduction: Population Without Age  Definitions of Rate of Increase	•
1.1	Doubling Time and Half-life The Period of Compounding. Application to Human History. Logarithms to Various Bases. Prospective Possible Doublings	3
1.2	One-Sex Versus Two-Sex Models: Descendants of the Pilgrim Fathers	9
1.3	How Many People Have Lived on the Earth?	12
1.4	A Mixture of Populations Having Different Rates of Increase An Arithmetic Example for Two Subpopulations	14
1.5	Rate of Increase Changing Over Time Special Cases of Changing Rates	18
1.6	Logistic Increase and Explosion	21
1.7	The Stalled Demographic Transition	23
1.8	Differential Fertility Due to the Demographic Transition	25
1.9	Matrices and Graphs in Demography A Two-Subgroup Model. Irreducibility or Connectivity. Primitivity. Application to Birth and Death	27
СНАР	TER 2	
The	Life Table	34
2.1	Definition of Life Table Functions  Mortality the Same for All Ages	34
2.2	Life Tables Based on Data Assuming Constant Probability of Dying Within the Age Interval. The Basic Equation and a Conventional Solution. A Precise Life Table	36
		xiii

xiv Contents

	Without Iteration or Graduation. Greville and Reed-Merrell Methods Derived as Special Cases. Bounds on Error	
2.3	Further Small Corrections	45
	Measure of Exposure	
2.4	Period and Cohort Tables	46
2.5	Financial Calculations	47
	Single-Payment Annuity and Insurance. Annual Premiums and Reserves	
2.6	Cause-Deleted Tables and Multiple Decrement	48
	Dependence of Causes of Death. Method of Calculation. Multiple Decrement	
2.7	The Life Table as a Unifying Technique in Demography	52
CHAP		
Mort	ality Comparisons; The Male-Female Ratio	54
	Variation by Age in the Sex Ratio of Mortality	
3.1	The Multiplicity of Index Numbers	.55
	Weighted Index of Male to Female Mortality. Aggregative Indices Versus Averages of Relatives	
3.2	Should We Index Death Rates or Survivorships?	60
3.3	Effect on $\ell_0$ of Change in $\mu(x)$	62
	A Proportional Difference Uniform at All Ages. Observed Values of the Constant $H$ . An Aspect of the Index Number Problem. Fractional Change in Mortality Due to a Given Cause. Comparison of $H^{(i)}$ with $\hat{e}_0^{(-i)} - \hat{e}_0$ . Interrelations of the Several Causes.	
3.4	Everybody Dies Prematurely	72
	Average Expectation of Life. Oldest Person in Group. Effect of a Health Improvement	
СНАР	TER 4	
Fixed	d Regime of Mortality and Fertility:	
The	Uses of Stable Theory	77
4.1	Stable Theory	78
	A Discrete Form	
4.2	Population Growth Estimated from One Census	81
	Effect of Choice of Model Life Table. Theory for the Error Arising from Use of an Improper Life Table	
4.3	Mean Age in the Stable Population	87
	Demographic Calculations Need Not Start at Age Zero. Use of Popula- tion Mean Age	

Contents	x
Contents	

4.4	Rate of Increase Estimated from the Fraction Under Age 25	92
4.5	Birth Rate as Well as Rate of Increase Estimated for a Stable Population	94
4.6	Comparison of the Several Ways of Using the Age Distribution	96
4.7	Incomplete Population and Deaths. Estimates from Two Censuses Sensitivity Analysis	103
,	Mean Age as a Function of Rate of Increase. Pension Cost. Fraction of Old People	100
4.8	The Degree to Which Promotion Within Organizations Depends on Population Increase A Simplification. The Chain Letter Principle	107
CHAPT		
Birth	and the Intrinsic Rate of Natural Increase	112
5.1	The Characteristic Equation Why Stress the Female Model? An Iterative Method for Calculating r. The Intrinsic Rate for Various Kinds of Data. Male Period Intrinsic Rates. Cohort Intrinsic Rate. Intrinsic Rate for One Family	113
5.2	A Variant Form of the Characteristic Equation	118
5.3	Perturbation Analysis of the Intrinsic Rate	120
	How the Intrinsic Rate Varies with the Moments. Change in Births at One Age	
5.4	Arbitrary Pattern of Birth Rate Decline Effect of Small Arbitrary Change in Birth Function. Amount of Change Needed for Drop to Bare Replacement. Effect of Uniformly Lower Death Rates	123
5.5	Drop in Births Required to Offset a Drop in Deaths The Drop in Fertility That Would Offset a Drop in Mortality to Zero. Diseases of Infancy Versus Heart Disease: Their Effects on Population Increase	126
5.6	Moments of the Dying Population in Terms of Those of the Living, and Conversely Expectation of Life as a Function of Crude Birth and Death Rates	129
5.7	Four Mathematical Formulations of the Basic Equation of Population The Lotka Integral Equation. The Leslie Matrix. The Difference Equation. The von Foerster Partial Differential Equations. The Four	133

xvi Contents

	TER 6	
_	oductive Value, with Applications to Migration,	
Cont	traception, and Zero Population Growth	142
6.1	Concept of Reproductive Value Reproductive Value from the Lotka Integral Equation. One Woman Aged x. Stable Age Distribution. Arbitrary Age Distribution. Numerical Calculation	143
6.2	Ultimate Effect of Small Out-migration Occurring in a Given Year	149
6.3	Effect of Continuing Birth Control and Sterilization	150
6.4	Large Change in Regime	15
6.5	Emigration as a Policy Applied Year After Year	153
6.6	The Momentum of Population Growth	15:
6.7	Eliminating Heart Disease Would Make Very Little Difference to Population Increase, Whereas Eradication of Malaria Makes a Great Deal of Difference	158
Appe	endix: Reproductive Value as a Contribution to Future Births	159
	TER 7 erstanding Population Characteristics	162
7.1	• •	163
7.1	Accounting for Age Distribution Young and Old Populations. Age Distribution as a Function of Rate of Increase. Neutral and Nonneutral Change in Mortality. Accounting for Observed Ages. Are Birth or Death Rates the Major Influence on Age Distribution?	103
7.2	Why There Are More Women Than Men at Older Ages in Modern Populations	170
7.3	The Stable Equivalent Population Projection and the Stable Approximation Thereto. Application of the Stable Equivalent Q. Relation Between Q and Reproductive Value V. A More General Stable Equivalent	172
7.4	Age at Marriage A Sum of Random Intervals Model. Small Marriage Circles. How Many Households Are Implied by Birth, Death, and Marriage Rates? Intrinsic Rates of Natural Increase: Age, Parity, and Nuptiality. The Life Cycle. Married and Divorced. The Current Divorce-Marriage Ratio	178
7.5	The Foreign-born and Internal Migrants	190

Conten	uts	xvi
7.6	Human Stocks and Flows	192
7.7	The Demography of Organizations Loss of Power. Organizing Political Success. Economic Hierarchies	197
	TER 8	20:
•	ection and Forecasting	201
8.1	Forecasting: Both Unavoidable and Impossible. Past Data, Present Action, and Future Conditions of Payoff Heavy Stakes on Simultaneous Lotteries. Projection as Distinct from Prediction	201
8.2	The Technique of Projection	205
	Survivorship. Reproduction. Extension to All Ages and Both Sexes. Age Versus Other Variables. Projection in a Heterogeneous Population	
8.3	Applications of Projection	212
	Population Dynamics with One Cause of Death Eliminated. Effect of Immediate Drop to Replacement Fertility	
8.4	The Search for Constancies	217
	Relational Methods. Mortality. Are Longitudinal Relations Demonstrated by Cross-Sectional Data?	
8.5	Features of Forecasting and Forecasting Error Extrapolation Versus Mechanism. Shape of the Projection Fan	221
8.6	The Components of Forecasting Error ex ante The Length of the Experience Base	227
8.7	Ex post Evaluation of Point Estimates	230
	Future Percentage Increase	
8.8	A Division of Labor	233
	The Loss Function Permits a Three-Way Division of Labor	
8.9	Interval Estimates as Currently Provided	235
	Official Agencies Have Backed into Confidence Intervals	
	TER 9	
Some	e Types of Instability	237
9.1	Absolute Change in Mortality the Same at All Ages Inferring the Increase in Births. Increase in Person-Years in Cohort	237
9.2	Proportional Change in Mortality	240
`	Rate of Increase of Births. Change of $\mathcal{E}_0$ . Increase in Total Cohort Population. Increase of Persons of Arbitrary Age	
9.3	Changing Birth Rates	243
9.4	Announced Period Birth Rate Too High	245

ents
2

9.5	Backward Population Projection Application	250
9.6	The Time to Stability The Criterion of Convergence. Use of the Characteristic Equation. An Exact Ratio of Partial Derivatives and an Approximation Thereto. Allowance for Different Ranges of Variance and Skewness Among Observed Populations. Time to Convergence. Theoretical Versus Empirical Relations	255
9.7	Retirement Pensions: Pay-as-You-Go Versus Actuarial Reserves	262
9.8	The Demography of Educational Organizations Under Changing Age Distributions	265
9.9	Two Levels of Students and Teachers	267
9.10	Mobility in an Unstable Population	269
9.11	The Easterlin Effect	270
	TER 10	
The	Demographic Theory of Kinship	273
10.1	Probability of Living Ancestors Counting Method. Probability Method. Living Mother by the Counting Method. Living Mother by Conditional Probability. Probability of Living Grandmother: Numerical Examples. Stable Results Versus a Kinship Census. An Approximation	275
10.2	Descendants	282
10.3	Sisters and Aunts A Paradox: The Average Girl Seems to Have Too Many Sisters. Age Incidence of Childbearing Conditional on Birth of One Child. Aunts	285
10.4	Mean and Variance of Ages Ascertainment	289
10.5	Generalization to Changing Rates of Birth and Death	291
10.6	Sensitivity Analysis Decomposition of $M_1(a)$ , the Probability of a Living Mother. Other Progenitors. Effect of Birth Pattern on Living Progenitors. Comparison of Effect of Birth and Death Rates	292
10.7	The Inverse Problem: Deriving Rates from Genealogies	299
10.8	Incest Taboo and Rate of Increase	300
10.9	The Bias Imposed by Age Difference on Cross-Cousin Marriage	301

xix

	TER 11 odemography	303
11.1	Births Averted by Contraception Abstention. Births Averted—the Causal Inference. Marginal Effect. Dropping the Contraceptive. Why 1000 Abortions Do Not Prevent 1000 Births in a Population. Abortion as a Backup to Contraception	303
11.2	Measurement of Fertility and Fecundity Probability of Conception by Days of the Month. Mean Fecundity from Surveys. Homogeneous Populations. A Heterogeneous Popula- tion with Fecundity Constant for Each Woman. The Pearl Index Is the Harmonic Mean of the Distribution. The Gini Fertility Measure. Comparison of Pearl and Gini Estimates. Excursus on Averages. Graduation Uses Information Efficiently. Mean and Variance Simul- taneously Estimated by Graduation. Life Table Methods for Fertility. Relation of Micro to Population Replacement. How Surer Contracep- tion Reduces the Interval Between Births	315
11.3	Why Three-Child Families Constitute a Population Explosion, Whereas Two-Child Families Would Lead to the Extinction of Mankind	330
11.4	A Family-Building Strategy to Avoid Extinction	332
11.5	Sex Preference and the Birth Rate An Approximation to the Harmonic Mean	335
11.6	Family-Building Strategy with Parental Control Over Sex of Children	338
11.7	Mean Family Size from Order-of-Birth Distribution	344
11.8	Parity Progression and Population Increase	345
11.9	For a Given Probability of Survivors, Lower Mortality Lowers the Rate of Increase	347
	TER 12 Multi-state Model	350
12.1	Single Decrement and Increment-Decrement	352
	Matrix of Inputs	JJ2
12.2	The Kolmogorov Equation The Multiplicative Property. Probabilities Over Long Intervals	355
12.3	Expected Time in the Several States Fertility Expectations	358
12.4	Projection	361

xx	Co	ontent
12.5	Transition Versus Instantaneous Probability of Moving	362
12.6	Stable Population	366
	TER 13	*
	lly Demography	368
13.1	Definitions Classifications. Theory and Statistical Compilation	369
13.2	Kinship	371
	Inference from Kin Counts. Widowhood. Theoretical Number of Families in the Population. Decomposing Widowhood	
13.3	The Life Cycle	376
	Shape of Family Tree. Headship	
13.4	Household Size Distribution Separate Living	379
13.5	Economic, Political and Biological Theory	381
13.6	Family Policy	383
-	TER 14 rogeneity and Selection in Population Analysis	385
	Historical Note	
14.1	Conditioning and the Interpretation of Statistical Data Simpson's Paradox	387
14.2	Heterogeneity and Selection	390
14.3	Application to Mortality	391
	A Mixture of Populations Having Different Rates of Increase. Two Classes of Frailty. Numerical Effect on Mortality. Frailty Varying Over Time	
14.4	Continuous Distribution of Increase and of Frailty Continuous Distribution of Frailty	395
14.5	Experimentation	399
CHAF	PTER 15	
Epilo	ogue: How Do We Know the Facts of Demography?	400
15.1	Growing Populations Have Smaller Proportions of Old People	402
	Older Population as a Function of Rate of Increase When All Else Is Constant. Are Births or Deaths Decisive?	
15.2	Promotion in Organizations	407
15.3	No Model, No Understanding	409

Contents		XX	
15.4	Too Many Models	410	
15.5	Effect of Development on Population Increase	411	
15.6	Effect of Population Growth on Development	412	
15.7	How Nature Covers Her Tracks The Oblique Use of Data to Challenge Theory	414	
15.8	The Psychology of Research	417	
Bibli	iography	419	
Index		435	

.