

Contents

Introduction	1
Part I	
Generalized Coherent States for the Simplest Lie Groups	
1. Standard System of Coherent States Related to the Heisenberg-Weyl Group: One Degree of Freedom	7
1.1 The Heisenberg-Weyl Group and Its Representations	8
1.1.1 The Heisenberg-Weyl Group	8
1.1.2 Representations of the Heisenberg-Weyl Group	11
1.1.3 Concrete Realization of the Representation $T^\lambda(g)$	12
1.2 Coherent States	13
1.3 The Fock-Bargmann Representation	19
1.4 Completeness of Coherent-State Subsystems	23
1.5 Coherent States and Theta Functions	26
1.6 Operators and Their Symbols	29
1.7 Characteristic Functions	37
2. Coherent States for Arbitrary Lie Groups	40
2.1 Definition of the Generalized Coherent State	40
2.2 General Properties of Coherent-State Systems	42
2.3 Completeness and Expansion in States of the CS System	43
2.4 Selection of Generalized CS Systems with States Closest to Classical	44
3. The Standard System of Coherent States; Several Degrees of Freedom	48
3.1 General Properties	48
3.2 Coherent States and Theta Functions for Several Degrees of Freedom	50
4. Coherent States for the Rotation Group of Three-Dimensional Space	54
4.1 Structure of the Groups $SO(3)$ and $SU(2)$	54
4.2 Representations of $SU(2)$	58
4.3 Coherent States	59

5. The Most Elementary Noncompact, Non-Abelian Simple Lie Group: $SU(1, 1)$	67
5.1 Group $SU(1, 1)$ and Its Representations	67
5.1.1 Fundamental Properties of $SU(1, 1)$	67
5.1.2 Discrete Series	70
5.1.3 Principal (Continuous) Series	71
5.2 Coherent States	73
5.2.1 Discrete Series	73
5.2.2 Principal (Continuous) Series	76
6. The Lorentz Group: $SO(3, 1)$	84
6.1 Representations of the Lorentz Group	84
6.2 Coherent States	86
7. Coherent States for the $SO(n, 1)$ Group: Class-I Representations of the Principal Series	93
7.1 Class-I Representations of $SO(n, 1)$	93
7.2 Coherent States	94
8. Coherent States for a Bosonic System with Finite Number of Degrees of Freedom	101
8.1 Canonical Transformations	101
8.2 Coherent States	104
8.3 Operators in the Space $\mathcal{H}_B^{(+)}$	107
9. Coherent States for a Fermionic System with Finite Number of Degrees of Freedom	111
9.1 Canonical Transformations	111
9.2 Coherent States	113
9.3 Operators in the Space $\mathcal{H}_F^{(+)}$	114

Part II

General Case

10. Coherent States for Nilpotent Lie Groups	119
10.1 Structure of Nilpotent Lie Groups	119
10.2 Orbits of Coadjoint Representation	120
10.3 Orbits of Nilpotent Lie Groups	121
10.4 Representations of Nilpotent Lie Groups	122
10.5 Coherent States	124
11. Coherent States for Compact Semisimple Lie Groups	126
11.1 Elements of the Theory of Compact Semisimple Lie Groups	126
11.2 Representations of Compact Simple Lie Groups	128
11.3 Coherent States	130

12. Discrete Series of Representations: The General Case	134
12.1 Discrete Series	134
12.2 Bounded Domains	135
12.3 Coherent States	139
13. Coherent States for Real Semisimple Lie Groups:	
Class-I Representations of Principal Series	145
13.1 Class-I Representations	145
13.2 Coherent States	147
13.3 Horocycles in Symmetric Space	148
13.4 Rank-1 Symmetric Spaces	149
13.5 Properties of Rank-1 CS Systems	152
13.6 Complex Homogeneous Bounded Domains	161
13.6.1 Type-I Tube Domains	164
13.6.2 Type-II Tube Domains	166
13.6.3 Type-III Tube Domains	167
13.6.4 Type-IV Domains	168
13.6.5 The Exceptional Domain D^V	170
13.7 Properties of the Coherent States	170
14. Coherent States and Discrete Subgroups:	
The Case of $SU(1, 1)$	173
14.1 Preliminaries	173
14.2 Incompleteness Criterion for CS Subsystems Related to Discrete Subgroups	174
14.3 Growth of a Function Analytical in a Disk Related to the Distribution of Its Zeros	176
14.4 Completeness Criterion for CS Subsystems	178
14.5 Discrete Subgroups of $SU(1, 1)$ and Automorphic Forms ...	179
15. Coherent States for Discrete Series and Discrete Subgroups:	
General Case	182
15.1 Automorphic Forms	182
15.2 Completeness of Some CS Subsystems	183
16. Coherent States and Berezin's Quantization	185
16.1 Classical Mechanics	186
16.2 Quantization	189
16.3 Quantization on the Lobachevsky Plane	191
16.3.1 Description of Operators	192
16.3.2 The Correspondence Principle	193
16.3.3 Operator T_h in Terms of a Laplacian	194
16.3.4 Representation of Group of Motions of the Lobachevsky Plane in Space $\mathcal{F}_h$	195

16.3.5 Quantization by Inversions Analog to Weyl Quantization	196
16.4 Quantization on a Sphere	197
16.5 Quantization on Homogeneous Kähler Manifolds	199

Part III

Physical Applications

17. Preliminaries	207
18. Quantum Oscillators	211
18.1 Quantum Oscillator Acted on by a Variable External Force .	211
18.2 Parametric Excitation of a Quantum Oscillator	213
18.3 Quantum Singular Oscillator	217
18.3.1 The Stationary Case	217
18.3.2 The Nonstationary Case	220
18.3.3 The Case of N Interacting Particles	222
18.4 Oscillator with Variable Frequency Acted on by an External Force	227
19. Particles in External Electromagnetic Fields	231
19.1 Spin Motion in a Variable Magnetic Field	231
19.2 Boson Pair Production in a Variable Homogeneous External Field	233
19.2.1 Dynamical Symmetry for Scalar Particles	233
19.2.2 The Multidimensional Case: Coherent States	237
19.2.3 The Multidimensional Case: Nonstationary Problem .	241
19.3 Fermion Pair Production in a Variable Homogeneous External Field	242
19.3.1 Dynamical Symmetry for Spin-1/2 particles	242
19.3.2 Heisenberg Representation	245
19.3.3 The Multidimensional Case: Coherent States	248
20. Generating Function for Clebsch-Gordan Coefficients of the $SU(2)$ group	253
21. Coherent States and the Quasiclassical Limit	256
22. $1/N$ Expansion for Gross-Neveu Models	260
22.1 Description of the Model	260
22.2 Dimensionality of Space $\mathcal{H}_N = \mathcal{F}_0$ in the Fermion Case	265
22.3 Quasiclassical Limit	266

23. Relaxation to Thermodynamic Equilibrium	270
23.1 Relaxation of Quantum Oscillator to Thermodynamic Equilibrium	270
23.1.1 Kinetic Equation	270
23.1.2 Characteristic Functions and Quasiprobability Distributions	271
23.1.3 Use of Operator Symbols	274
23.2 Relaxation of a Spinning Particle to Thermodynamic Equilibrium in the Presence of a Magnetic Field	278
24. Landau Diamagnetism	282
25. The Heisenberg-Euler Lagrangian	286
26. Synchrotron Radiation	289
27. Classical and Quantal Entropy	292
Appendix A. Proof of Completeness for Certain CS Subsystems	296
Appendix B. Matrix Elements of the Operator $D(\gamma)$	300
Appendix C. Jacobians of Group Transformations for Classical Domains	305
Further Applications of the CS Method	308
References	311
Subject-Index	319
Addendum. Further Applications of the CS Method	308
References	311
References to Addendum	316
Subject-Index	319