Contents

ΙT	he Problem of Formulating an Axiomatics for Quantum Mechanics .	. 1
§ 2	Is There an Axiomatic Basis for Quantum Mechanics?	. 2
§ 3	Experimental Situations Describable Solely by Pretheories	. 3
§ 4	Mathematical Problems	. 5
§ 5	Progress to More Comprehensive Theories	. 6
II I	Pretheories for Quantum Mechanics	. 8
	State Space and Trajectory Space	
§ 2	Preparation and Registration Procedures	. 17
	§ 2.1 Statistical Selection Procedures	
	§ 2.2 Preparation Procedures	
	§ 2.3 Registration Procedures	
	§ 2.4 Dependence of Registration on Preparation	
§ 3	Trajectory Preparation and Registration Procedures	
	§ 3.1 Trajectory Effects	. 26
	§ 3.2 Trajectory Ensembles	. 30
	§ 3.3 The Dynamic Laws and the Objectivating Manner of Description	
	§ 3.4 Dynamically Continuous Systems	
§ 4	Transformations of Preparation and Registration Procedures	
	§ 4.1 Time Translations of the Trajectory Registration Procedures	. 37
	§ 4.2 Time Translations of the Preparation Procedures	. 40
	§ 4.3 Further Transformations of Preparation	
	and Registration Procedures	. 42
§ 5	The Macrosystems as Physical Objects	. 43
Ш	Base Sets and Fundamental Structure Terms	
	for a Theory of Microsystems	. 47
§ 1	Composite Macrosystems	. 47
§ 2	Preparation and Registration Procedures for Composite Macrosystems	50
§ 3	Directed Interactions	. 59
§ 4	Action Carriers	. 68
§ 5	Ensembles and Effects	. 74
-	§ 5.1 The Problem of Combining Preparation	
	and Registration Procedures	. 74



VIII Contents

	§ 5.2 Physical Systems	79
	§ 5.3 Mixing and De-mixing of Ensembles and Effects	81
	§ 5.4 Re-elimination of the Action Carrier	85
§ 6	Objectivating Method of Describing Experiments	86
	§ 6.1 The Method of Describing Composite Macrosystems	
	in the Trajectory Space	87
	§ 6.2 Trajectory Effects of the Composite Systems	87
	§ 6.3 Trajectory Ensembles of the Composite Systems	89
	§ 6.4 The Structure of the Trajectory Measures	
	for Directed Action	92
	§ 6.5 Complete Description by Trajectories	94
	§ 6.6 Use of the Interaction for the Registration of Macrosystems	97
	§ 6.7 The Relation Between the Two Forms of an Axiomatic Basis	98
8 7	Transport of Systems Relative to Each Other	99
3 ′	Transport of Systems relative to Each Other	,,
IV	Embedding of Ensembles and Effect Sets in Topological Vector Spaces	101
-	Embedding of \mathcal{H} , \mathcal{L} in a Dual Pair of Vector Spaces	101
	Uniform Structures of the Physical Imprecision on \mathscr{K} and \mathscr{L}	103
§ 3	Embedding of \mathcal{H} and \mathcal{L} in Topologically Complete Vector Spaces	104
§ 4	$\mathscr{B}, \mathscr{B}', \mathscr{D}, \mathscr{D}'$ Considered as Ordered Vector Spaces	114
§ 5	The Faces of K and L	119
§ 6	Some Convergence Theorems	123
V	Observables and Preparators	126
8 1	Coexistent Effects and Observables	126
3 1	§ 1.1 Coexistent Registrations	126
	§ 1.2 Coexistent Effects	127
	§ 1.3 Observables	128
8 2	Mixture Morphisms	129
	Structures in the Class of Observables	131
g J	§ 3.1 The Spaces $\mathscr{B}(\Sigma)$ and $\mathscr{B}'(\Sigma)$ Assigned to a Boolean Ring Σ	131
	§ 3.2 Mixture Morphism Corresponding to an Observable	131
	§ 3.3 The Kernel of an Observable	132
	§ 3.4 De-mixing of Observables	134
		134
	§ 3.5 Measurement Scales of Observables	126
	and Totally Ordered Subsets of L	136
	Coexistent and Complementary Observables	137
•	Realization of Observables	137
	Coexistent De-mixing of Ensembles	140
-	Complementary De-mixings of Ensembles	143
	Realizations of De-mixings	145
	Preparators and Faces of K	146
§ 1	0 Physical Objects as Action Carriers	148
§ 1	1 Operations and Transpreparators	150

Contents

IX

VI.	Main Laws of Preparation and Registration				•	152
§ 1	Main Laws for the Increase in Sensitivity of Registrations					153
Ū	§ 1.1 Increase in Sensitivity Relative to Two Effect Procedures					153
	§ 1.2 Some Experimental and Intuitive Indications for					
	the Law of Increase in Sensitivity					156
	§ 1.3 Decision Effects					159
	§ 1.4 The Increase in Sensitivity of an Effect					162
§ 2	Relations Between Preparation and Registration Procedures					164
0 –	§ 2.1 Main Law for the De-mixing of Ensembles and					
	Related Possibilities of Registering					164
	§ 2.2 Some Consequences of Axiom AV2					166
§ 3	The Lattice G					167
	Commensurable Decision Effects					172
	The Orthomodularity of G					174
	The Main Law for Not Coexistent Registrations					176
3 0	§ 6.1 Experimental Hints for Formulating the Main Law	•			•	
	for Not Coexistent Registrations					176
	§ 6.2 Some Important Equivalences					180
	§ 6.3 Formulation of the Main Law and Some Consequences					182
8 7	The Main Law of Quantization					184
3 '	§ 7.1 Intuitive Indications for Formulating	•	•	•	•	
	the Main Law of Quantization				_	184
	§ 7.2 Simple Consequences of the Main Law of Quantization .					186
	3 7.2 Simple consequences of the Main Earl of Quantization .	•	•	•	•	100
VII	Decision Observables and the Center					187
§ 1	The Commutator of a Set of Decision Effects				•	187 187
§ 1					•	
§ 1 § 2	The Commutator of a Set of Decision Effects					187
§ 1 § 2	The Commutator of a Set of Decision Effects					187 188
§ 1 § 2 § 3	The Commutator of a Set of Decision Effects					187 188
§ 1 § 2 § 3	The Commutator of a Set of Decision Effects					187 188 189
§ 1 § 2 § 3	The Commutator of a Set of Decision Effects Decision Observables					187 188 189 190
§ 1 § 2 § 3	The Commutator of a Set of Decision Effects Decision Observables					187 188 189 190 190
§ 1 § 2 § 3	The Commutator of a Set of Decision Effects					187 188 189 190 190
§ 1§ 2§ 3§ 4§ 5	The Commutator of a Set of Decision Effects					187 188 189 190 190 191 193
§ 1§ 2§ 3§ 4§ 5	The Commutator of a Set of Decision Effects					187 188 189 190 190 191 193 199
§ 1 § 2 § 3 § 4 § 5	The Commutator of a Set of Decision Effects					187 188 189 190 190 191 193 199 202
\$ 1 \$ 2 \$ 3 \$ 4 \$ 5	The Commutator of a Set of Decision Effects Decision Observables Structures in That Class of Observables Whose Range also Contains Elements of G Commensurable Decision Observables Decomposition of And A'Relative to the Center Z § 5.1 Reduction of the Elements of A' by the Elements of G § 5.2 Reduction by Center Elements § 5.3 Classical Systems § 5.4 Decomposition into Irreducible Parts System Types and Super Selection Rules I Representation of A, A' by Banach Spaces of Operators in a Hilbert Space					187 188 189 190 191 193 199 202 205
\$ 1 \$ 2 \$ 3 \$ 4 \$ 5 VII	The Commutator of a Set of Decision Effects Decision Observables Structures in That Class of Observables Whose Range also Contains Elements of G Commensurable Decision Observables Decomposition of A and A'Relative to the Center Z § 5.1 Reduction of the Elements of A' by the Elements of G § 5.2 Reduction by Center Elements § 5.3 Classical Systems § 5.4 Decomposition into Irreducible Parts System Types and Super Selection Rules I Representation of A, A' by Banach Spaces of Operators in a Hilbert Space The Finite Elements of G					187 188 189 190 191 193 199 202 205 208
\$ 1 \$ 2 \$ 3 \$ 4 \$ 5 VII \$ 1 \$ 2	The Commutator of a Set of Decision Effects Decision Observables Structures in That Class of Observables Whose Range also Contains Elements of G Commensurable Decision Observables Decomposition of And Arelative to the Center Z § 5.1 Reduction of the Elements of Break by the Elements of G § 5.2 Reduction by Center Elements § 5.3 Classical Systems § 5.4 Decomposition into Irreducible Parts System Types and Super Selection Rules I Representation of A, Break by Banach Spaces of Operators in a Hilbert Space The Finite Elements of G The General Representation Theorem for Irreducible G					187 188 189 190 191 193 199 202 205 208 208 210
\$ 1 \$ 2 \$ 3 \$ 4 \$ 5 VII \$ 1 \$ 2 \$ 3	The Commutator of a Set of Decision Effects Decision Observables Structures in That Class of Observables Whose Range also Contains Elements of G Commensurable Decision Observables Decomposition of and are Relative to the Center Z § 5.1 Reduction of the Elements of by the Elements of G § 5.2 Reduction by Center Elements § 5.3 Classical Systems § 5.4 Decomposition into Irreducible Parts System Types and Super Selection Rules I Representation of a, by by Banach Spaces of Operators in a Hilbert Space The Finite Elements of G The General Representation Theorem for Irreducible G Some Topological Properties of G					187 188 189 190 191 193 199 202 205 208 208 210 212
\$ 1 \$ 2 \$ 3 \$ 4 \$ 5 VII \$ 1 \$ 2 \$ 3	The Commutator of a Set of Decision Effects Decision Observables Structures in That Class of Observables Whose Range also Contains Elements of G Commensurable Decision Observables Decomposition of And R'Relative to the Center Z § 5.1 Reduction of the Elements of B' by the Elements of G § 5.2 Reduction by Center Elements § 5.3 Classical Systems § 5.4 Decomposition into Irreducible Parts System Types and Super Selection Rules I Representation of B, B' by Banach Spaces of Operators in a Hilbert Space The Finite Elements of G The General Representation Theorem for Irreducible G Some Topological Properties of G The Representation Theorem for K, L					187 188 189 190 191 193 199 202 205 208 208 210 212 215
\$ 1 \$ 2 \$ 3 \$ 4 \$ 5 VII \$ 1 \$ 2 \$ 3	The Commutator of a Set of Decision Effects Decision Observables Structures in That Class of Observables Whose Range also Contains Elements of G Commensurable Decision Observables Decomposition of and are Relative to the Center Z § 5.1 Reduction of the Elements of by the Elements of G § 5.2 Reduction by Center Elements § 5.3 Classical Systems § 5.4 Decomposition into Irreducible Parts System Types and Super Selection Rules I Representation of a, by by Banach Spaces of Operators in a Hilbert Space The Finite Elements of G The General Representation Theorem for Irreducible G Some Topological Properties of G					187 188 189 190 191 193 199 202 205 208 208 210 212

X Contents

§ 4.3 Coexistence, Commensurability, Uncertainty Relations,	
and Commutability of Operators	221
§ 5 Some Theorems for Finite-dimensional and Irreducible @	223
Appendix	227
A I Some Theorems for Atoms in the Lattice G	227
A II Banach Lattices	229
A III The Axiom AVid and the Minimal Decomposition Property	230
A IV The Bishop-Phelps Theorem and the Ellis Theorem	231
Bibliography	235
List of Frequently Used Symbols	239
List of Axioms	241
Indov	242