

Table of Contents

Chapter 1. Introduction	1
1.1 General Introduction	1
1.2 Dissipative Structures in Physical, Chemical, and Biological Systems	3
1.2.1 The problems of elastic stability	3
1.2.2 Bifurcations to divergence and flutter in flow-induced oscillations	5
1.2.3 Wheelset nonlinear hunting problem	5
1.2.4 Buckling of a shallow elastic arch	6
1.2.5 Dissipative structures in fluid mechanics	6
1.2.6 Biological systems	10
1.2.7 Reaction-diffusion problems	15
1.3 Basic Concepts and Properties of Nonlinear Systems	18
1.3.1 Steady-state solutions	19
1.3.2 Stability of solutions	23
1.3.3 Evolution systems	27
1.4 Examples	27
 Chapter 2. Multiplicity and Stability in Lumped-Parameter Systems (LPS)	 36
2.1 Steady-State Solutions	36
2.2 Dependence of Steady-State Solutions on a Parameter—Solution Diagram	39
2.3 Stability of Steady-State Solutions	47
2.4 Branch Points—Real Bifurcation	49
2.4.1 Evaluation of limit and bifurcation points	51
2.4.2 Direction of branches at a bifurcation point	56
2.4.2.1 Selecting starting points for the continuation algorithm	60
2.4.2.2 Illustrative examples	62
2.4.2.3 Bifurcation points with higher degeneration	64
2.4.3 Occurrence of isolas, isola formation	65
2.5 Branch Points—Complex Bifurcations	69
2.5.1 The Hopf bifurcation theorem	70
2.5.2 Direct decomposition technique for location of the complex bifurcation point	72
2.5.3 Direct iteration techniques	76

2.6	Bifurcation Diagram	80
2.7	Transient Behavior of LPS—Numerical Methods.	83
2.7.1	Runge–Kutta methods	84
2.7.2	Multistep methods	85
2.7.3	Integration along the solution arc	86
2.7.4	Integration of phase trajectories for autonomous systems	87
2.7.5	Numerical methods for stiff systems of ODE	87
2.7.6	Systems of differential and algebraic equations	90
2.7.7	Integration of differential equations with time delay	91
2.8	Computation of Periodic Solutions	92
2.8.1	Transformation into an initial-value problem—the shooting method	94
2.8.2	Stability of periodic solutions	95
2.8.3	Continuation of periodic solutions	97
2.8.4	Bifurcation of periodic solutions	99
2.9	Chaotic Attractors	101
2.9.1	Characterization of chaotic attractors	103
2.9.2	Liapunov exponents	103
2.9.3	Power spectra	109
2.9.4	The Poincaré map	111

Chapter 3. Multiplicity and Stability in Distributed-Parameter Systems (DPS) 114

3.1	Steady-State Solutions—Methods for Solving Nonlinear Boundary-Value Problems	116
3.1.1	Finite-difference methods	116
3.1.2	Quasi-linearization	119
3.1.3	Shooting methods	119
3.2	Dependence of Steady-State Solutions on a Parameter	122
3.3	Branch Points—Methods for Evaluating Real and Complex Bifurcation Points	133
3.3.1	Primary bifurcation	134
3.3.2	Secondary real bifurcation	138
3.3.3	Secondary complex bifurcation	146
3.4	Methods for Transient Simulation of Parabolic Equations—Finite-Difference Methods	151
3.4.1	Nonlinearity approximation	153
3.4.2	Automatic control of time step k	154
3.4.3	Automatic control of spatial step size h , equidistant net	155
3.4.4	Adaptive nonequidistant net	156

Chapter 4. Development of Quasi-stationary Patterns with Changing Parameter 159

4.1	Quasi-stationary Behavior in LPS—Examples	162
4.2	Quasi-stationary Behavior in DPS—Examples	166

Chapter 5. Perspectives	175
Appendix A DERPAR—A Continuation Algorithm	183
Appendix B SHOOT—An Algorithm for Solving Nonlinear Boundary-Value Problems by the Shooting Method	187
Appendix C Bifurcation and Stability Theory	196
C.1 Invariant Manifolds and the Center-Manifold Theorem (Reduction of Dimension)	197
C.2 Normal Forms	198
C.3 Bifurcation of Singular Points of Vector Fields	207
C.4 Codimension of a Vector Field. Unfolding of a Vector Field.	208
C.5 Construction of a Versal Deformation	214
C.6 Bifurcations of Codimension 2	217
C.7 Bifurcations from Limit Cycles	219
References	233
Index	241