

<i>Chapter 1</i>	<i>APPROXIMATE METHODS</i>	1
1.1.	Introduction	1
1.2.	Basic Definitions	2
1.3.	Approximate Solutions	7
1.4.	Method of Weighted Residuals	12
1.4.1.	The Collocation Method	13
1.4.2.	Method of Collocation by Subregions	17
1.5.	Method of Galerkin	23
1.6.	Weak Formulations	25
1.7.	Inverse Problem and Boundary Solutions	35
1.8.	Classification of Approximate Methods	43
References		44
Bibliography		45
 <i>Chapter 2</i>	 <i>POTENTIAL PROBLEMS</i>	 47
2.1.	Introduction	47
2.2.	Elements of Potential Theory	49
2.3.	Indirect Formulation	58
2.4.	Direct Formulation	61
2.5.	Boundary Element Method	64
2.6.	Two-Dimensional Problems	65
2.6.1.	Source Formulation	70
2.7.	Poisson Equation	75
2.8.	Subregions	79
2.9.	Orthotropy and Anisotropy	82
2.10.	Infinite Regions	85
2.11.	Special Fundamental Solutions	89
2.12.	Three-Dimensional Problems	92
2.13.	Axisymmetric Problems	96
2.14.	Axisymmetric Problems with Arbitrary Boundary Conditions	99
2.15.	Nonlinear Materials and Boundary Conditions	102
2.15.1.	Nonlinear Boundary Conditions	106
References		107
 <i>Chapter 3</i>	 <i>INTERPOLATION FUNCTIONS</i>	 109
3.1.	Introduction	109
3.2.	Linear Elements for Two-Dimensional Problems	109

3.3.	Quadratic and Higher-Order Elements	118
3.4.	Boundary Elements for Three-Dimensional Problems	127
3.4.1.	Quadrilateral Elements	129
3.4.2.	Higher-Order Quadrilateral Elements	131
3.4.3.	Lagrangian Quadrilateral Elements	131
3.4.4.	Triangular Elements	132
3.4.5.	Higher-Order Triangular Elements	134
3.5.	Three-Dimensional Cell Elements	135
3.5.1.	Tetrahedron	136
3.5.2.	Cube	136
3.6.	Discontinuous Boundary Elements	137
3.7.	Order of Interpolation Functions	138
	References	140

Chapter 4 DIFFUSION PROBLEMS 141

4.1.	Introduction	141
4.2.	Laplace Transforms	142
4.3.	Coupled Boundary Element – Finite Difference Methods	146
4.4.	Time-Dependent Fundamental Solutions	147
4.5.	Two-Dimensional Problems	150
4.5.1.	Constant Time Interpolation	150
4.5.2.	Linear Time Interpolation	152
4.5.3.	Quadratic Time Interpolation	153
4.5.4.	Space Integration	154
4.6.	Time-Marching Schemes	156
4.7.	Three-Dimensional Problems	164
4.8.	Axisymmetric Problems	165
4.9.	Nonlinear Diffusion	171
	References	174

Chapter 5 ELASTOSTATICS 177

5.1.	Introduction to the Theory of Elasticity	177
5.1.1.	Initial Stresses or Initial Strains	183
5.2.	Fundamental Integral Statement	183
5.2.1.	Somigliana Identity	185
5.3.	Fundamental Solutions	187
5.4.	Stresses at Internal Points	190
5.5.	Boundary Integral Equation	191
5.6.	Infinite and Semi-Infinite Regions	195
5.7.	Numerical Implementation	197
5.8.	Boundary Elements	199
5.9.	System of Equations	201
5.10.	Stresses and Displacements Inside the Body	202
5.11.	Stresses on the Boundary	203
5.12.	Surface Traction Discontinuities	204

5.13. Two-Dimensional Elasticity	210
5.14. Body Forces	217
5.14.1. Gravitational Loads	219
5.14.2. Centrifugal Load	220
5.14.3. Thermal Loading	222
5.15. Axisymmetric Problems	224
5.15.1. Extension to Nonaxisymmetric Boundary Values	230
5.16. Anisotropy	230
References	234

Chapter 6

BOUNDARY INTEGRAL FORMULATION FOR INELASTIC PROBLEMS

237

6.1. Introduction	237
6.2. Inelastic Behavior of Materials	240
6.3. Governing Equations	251
6.4. Boundary Integral Formulation	253
6.5. Internal Stresses	255
6.6. Alternative Boundary Element Formulations	258
6.6.1. Initial Strain	258
6.6.2. Initial Stress	260
6.6.3. Fictitious Traction and Body Forces	261
6.7. Half-Plane Formulations	262
6.8. Spatial Discretization	265
6.9. Internal Cells	270
6.10. Axisymmetric Case	274
References	275

Chapter 7

ELASTOPLASTICITY

277

7.1. Introduction	277
7.2. Some Simple Elastoplastic Relations	277
7.3. Initial Strain: Numerical Solution Technique	281
7.3.1. Examples – Initial Strain Formulation	282
7.4. General Elastoplastic Stress-Strain Relations	286
7.5. Initial Stress: Outline of Solution Techniques	290
7.5.1. Examples: Kelvin Implementation	292
7.5.2. Examples: Half-Plane Implementation	297
7.6. Comparison with Finite Elements	300
References	304

Chapter 8

OTHER NONLINEAR MATERIAL PROBLEMS

306

8.1. Introduction	306
8.2. Rate-Dependent Constitutive Equations	306
8.3. Solution Technique: Viscoplasticity	309

8.4. Examples: Time-Dependent Problems	312
8.5. No-Tension Materials	318
References	322

Chapter 9

PLATE BENDING

324

9.1. Introduction	324
9.2. Governing Equations	324
9.3. Integral Equations	326
9.3.1. Other Fundamental Solutions	330
9.4. Applications	331
References	336

Chapter 10

WAVE PROPAGATION PROBLEMS

338

10.1. Introduction	338
10.2. Three-Dimensional Water Wave Propagation Problems	339
10.3. Vertical Axisymmetric Bodies	344
10.4. Horizontal Cylinders of Arbitrary Section	347
10.5. Vertical Cylinders of Arbitrary Section	350
10.6. Transient Scalar Wave Equation	352
10.7. Three-Dimensional Problems: The Retarded Potential	354
10.8. Two-Dimensional Problems	356
References	357

Chapter 11

VIBRATIONS

360

11.1. Introduction	360
11.2. Governing Equations	360
11.3. Time-Dependent Integral Formulation	362
11.4. Laplace Transform Formulation	363
11.5. Steady-State Elastodynamics	367
11.6. Free Vibrations	373
References	375

Chapter 12

FURTHER APPLICATIONS IN
FLUID MECHANICS

377

12.1. Introduction	377
12.2. Transient Groundwater Flow	377
12.3. Moving Interface Problems	381
12.4. Axisymmetric Bodies in Cross Flow	384
12.5. Slow Viscous Flow (Stokes Flow)	386
12.6. General Viscous Flow	389
12.6.1. Steady Problems	393
12.6.2. Transient Problems	395
References	398

<i>Chapter 13</i>	<i>COUPLING OF BOUNDARY ELEMENTS WITH OTHER METHODS</i>	400
13.1.	Introduction	400
13.2.	Coupling of Finite Element and Boundary Element Solutions . . .	401
13.2.1.	The Energy Approach	405
13.3.	Alternative Approach	409
13.4.	Internal Fluid Problems	411
13.4.1.	Free-Surface Boundary Condition	412
13.4.2.	Extension to Compressible Fluid	414
13.5.	Approximate Boundary Elements	415
13.6.	Approximate Finite Elements	422
References		424
 <i>Chapter 14</i>	 <i>COMPUTER PROGRAM FOR TWO-DIMENSIONAL ELASTOSTATICS</i>	 427
14.1.	Introduction	427
14.2.	Main Program and Data Structure	428
14.3.	Subroutine INPUT	430
14.4.	Subroutine MATRX	433
14.5.	Subroutine FUNC	435
14.6.	Subroutine SLNPD	437
14.7.	Subroutine OUTPT	438
14.8.	Subroutine FENC	439
14.9.	Examples	440
14.9.1.	Square Plate	440
14.9.2.	Cylindrical Cavity Problem	446
References		446
 <i>Appendix A</i>	 <i>NUMERICAL INTEGRATION FORMULAS</i>	 447
A.1.	Introduction	447
A.2.	Standard Gaussian Quadrature	447
A.2.1.	One-Dimensional Quadrature	447
A.2.2.	Two- and Three-Dimensional Quadrature for Rectangles and Rectangular Hexahedra	447
A.2.3.	Triangular Domain	449
A.3.	Computation of Singular Integrals	449
A.3.1.	One-Dimensional Logarithmic Gaussian Quadrature Formulas	449
A.3.2.	Numerical Integration over Triangles and Squares with $1/r$ Singularity	449
A.3.3.	Numerical Evaluation of Cauchy Principal Values	451
References		454

<i>Appendix B</i>	<i>SEMI-INFINITE FUNDAMENTAL SOLUTIONS</i>	. .	455
B.1. Half-Space			455
B.2. Half-Plane			458
References			460
<i>Appendix C</i>	<i>SOME PARTICULAR EXPRESSIONS FOR TWO-DIMENSIONAL INELASTIC PROBLEMS</i>	. . .	461
<i>SUBJECT INDEX</i>			463