

Contents

CHAPTER 1

Preliminary Material	1
1.1. Notation	1
1.2. Inequalities	2
1.3. Algebra	3
1.4. Topology	3
1.5. Topological Groups	5
1.6. Analysis	6

CHAPTER 2

Matrices	9
2.1. Non-singular Matrices	9
2.2. The Metric Structure	11
2.3. Discrete Groups	14
2.4. Quaternions	16
2.5. Unitary Matrices	17

CHAPTER 3

Möbius Transformations on $\mathbb{R}^n$	20
3.1. The Möbius Group on $\mathbb{R}^n$	20
3.2. Properties of Möbius Transformations	28
3.3. The Poincaré Extension	33
3.4. Self-mappings of the Unit Ball	36
3.5. The General Form of a Möbius Transformation	40
3.6. Distortion Theorems	42
3.7. The Topological Group Structure	45
3.8. Notes	54

CHAPTER 4

Complex Möbius Transformations	56
4.1. Representations by Quaternions	56
4.2. Representation by Matrices	60
4.3. Fixed Points and Conjugacy Classes	64
4.4. Cross Ratios	75
4.5. The Topology on $\mathcal{M}$	78
4.6. Notes	82

CHAPTER 5

Discontinuous Groups	83
5.1. The Elementary Groups	83
5.2. Groups with an Invariant Disc	92
5.3. Discontinuous Groups	94
5.4. Jørgensen's Inequality	104
5.5. Notes	115

CHAPTER 6

Riemann Surfaces	116
6.1. Riemann Surfaces	116
6.2. Quotient Spaces	117
6.3. Stable Sets	122

CHAPTER 7

Hyperbolic Geometry	126
----------------------------	-----

Fundamental Concepts

7.1. The Hyperbolic Plane	126
7.2. The Hyperbolic Metric	129
7.3. The Geodesics	134
7.4. The Isometries	136
7.5. Convex Sets	138
7.6. Angles	141

Hyperbolic Trigonometry

7.7. Triangles	142
7.8. Notation	144
7.9. The Angle of Parallelism	145
7.10. Triangles with a Vertex at Infinity	146
7.11. Right-angled Triangles	146
7.12. The Sine and Cosine Rules	148
7.13. The Area of a Triangle	150
7.14. The Inscribed Circle	151

Polygons

7.15. The Area of a Polygon	153
7.16. Convex Polygons	154
7.17. Quadrilaterals	156
7.18. Pentagons	159
7.19. Hexagons	160

The Geometry of Geodesics

7.20. The Distance of a Point from a Line	162
7.21. The Perpendicular Bisector of a Segment	164
7.22. The Common Orthogonal of Disjoint Geodesics	165
7.23. The Distance Between Disjoint Geodesics	166
7.24. The Angle Between Intersecting Geodesics	166
7.25. The Bisector of Two Geodesics	166
7.26. Transversals	167

Pencils of Geodesics

7.27. The General Theory of Pencils	168
7.28. Parabolic Pencils	169
7.29. Elliptic Pencils	170
7.30. Hyperbolic Pencils	170

The Geometry of Isometries

7.31. The Classification of Isometries	171
7.32. Parabolic Isometries	172
7.33. Elliptic Isometries	172
7.34. Hyperbolic Isometries	173
7.35. The Displacement Function	174
7.36. Isometric Circles	176
7.37. Canonical Regions	177
7.38. The Geometry of Products of Isometries	179
7.39. The Geometry of Commutators	184
7.40. Notes	187

CHAPTER 8

Fuchsian Groups	188
8.1. Fuchsian Groups	188
8.2. Purely Hyperbolic Groups	190
8.3. Groups Without Elliptic Elements	198
8.4. Criteria for Discreteness	200
8.5. The Nielsen Region	202
8.6. Notes	203

CHAPTER 9

Fundamental Domains	204
9.1. Fundamental Domains	204
9.2. Locally Finite Fundamental Domains	206
9.3. Convex Fundamental Polygons	217
9.4. The Dirichlet Polygon	226
9.5. Generalized Dirichlet Polygons	234
9.6. Fundamental Domains for Coset Decompositions	238
9.7. Side-Pairing Transformations	240
9.8. Poincaré's Theorem	242
9.9. Notes	252

CHAPTER 10

Finitely Generated Groups	253
10.1. Finite Sided Fundamental Polygons	253
10.2. Points of Approximation	258
10.3. Conjugacy Classes	263
10.4. The Signature of a Fuchsian Group	268
10.5. The Number of Sides of a Fundamental Polygon	274
10.6. Triangle Groups	276
10.7. Notes	286

CHAPTER 11

Universal Constraints on Fuchsian Groups	287
11.1. Uniformity of Discreteness	287
11.2. Universal Inequalities for Cycles of Vertices	288
11.3. Hecke Groups	293
11.4. Trace Inequalities	295
11.5. Three Elliptic Elements of Order Two	301
11.6. Universal Bounds on the Displacement Function	308
11.7. Canonical Regions and Quotient Surfaces	324
11.8. Notes	327

References	329
-------------------	-----

Index	335
--------------	-----