Frank W. Warner

FOUNDATIONS of DIFFERENTIABLE MANIFOLDS and LIE GROUPS

With 57 Illustrations



Contents

1 MANIFOLDS

- 2 Preliminaries
- 5 Differentiable Manifolds
- 8 The Second Axiom of Countability
- 11 Tangent Vectors and Differentials
- 22 Submanifolds, Diffeomorphisms, and the Inverse Function Theorem
- 30 Implicit Function Theorems
- 34 Vector Fields
- 41 Distributions and the Frobenius Theorem
- 50 Exercises

2 tensors and differential forms

- 54 Tensor and Exterior Algebras
- 62 Tensor Fields and Differential Forms
- 69 The Lie Derivative
- 73 Differential Ideals
- 77 Exercises

3 LIE GROUPS

- 82 Lie Groups and Their Lie Algebras
- 89 Homomorphisms
- 92 Lie Subgroups
- 98 Coverings
- 101 Simply Connected Lie Groups
- 102 Exponential Map
- 109 Continuous Homomorphisms
- 110 Closed Subgroups
- 112 The Adjoint Representation
- 117 Automorphisms and Derivations of Bilinear Operations and Forms
- 120 Homogeneous Manifolds
- 132 Exercises

	INTEGRATION ON MANIFOLDS
138 140 153 157	Orientation Integration on Manifolds de Rham Cohomology Exercises
5	SHEAVES, COHOMOLOGY, AND THE DE RHAM THEOREM
163 173 176 186 189 191 200 205 207 214 216	Sheaves and Presheaves Cochain Complexes Axiomatic Sheaf Cohomology The Classical Cohomology Theories Alexander-Spanier Cohomology de Rham Cohomology Singular Cohomology Čech Cohomology The de Rham Theorem Multiplicative Structure Supports Exercises
6	THE HODGE THEOREM
220 222 227 240 243 250 251	The Laplace-Beltrami Operator The Hodge Theorem Some Calculus Elliptic Operators Reduction to the Periodic Case Ellipticity of the Laplace-Beltrami Operator Exercises
260	BIBLIOGRAPHY
262	SUPPLEMENT TO THE BIBLIOGRAPHY
264	INDEX OF NOTATION
267	INDEX