

Contents

0. Introduction	1
I. An Introduction to Point Processes	9
1. Point Processes	11
1.1. Point Processes	12
1.1.1. Basic Notions	12
1.1.2. Moment Measures and Functionals of Point Processes	16
1.1.3. The Poisson Process	18
1.2. The Campbell Measure of a Point Process	23
1.2.1. Disintegration with respect to the Intensity Measure: Palm Distributions	24
1.2.2. Disintegration with respect to the Point Process: Pa- pangelou Kernels	26
1.2.3. Construction of Point Processes from Papangelou Ker- nels	27
2. Deviation Principles	39
2.1. General Large Deviation Principles	40
2.2. Large Deviations for Poisson Processes at increasing Inten- sity	42
2.3. Deviations for Brownian Motions and Brownian Bridges . .	45

II. A Bose Gas Model	49
3. Construction of the ideal Bose Gas	51
3.1. The Loop Space and the Brownian Loop Measure	52
4. Limit theorems and Extremal Measures	63
4.1. The Construction of Martin-Dynkin Boundaries	66
4.1.1. Local Specifications and Martin-Dynkin Boundary .	66
4.1.2. Counting Loops	67
4.2. The Microcanonical Loop Ensemble	69
4.3. The Canonical Loop Ensemble	75
4.4. The Grand Canonical Loop Ensemble	78
4.5. The Canonical Ensemble of Elementary Components . . .	79
5. Geometric Aspects of the ideal Bose Gas	91
5.1. Palm Distributions and Stationarity	94
5.2. The Barycentre	98
5.2.1. The Barycentre of a Brownian Loop	98
5.2.2. The Barycentre of a Random Walk Loop	102
5.3. k -Volumes	108
5.3.1. k -Volumes of independent Vectors	108
5.3.2. k -Volumes of dependent Vectors	110
5.3.3. k -Volumes of Random Walk Loops	112
5.3.4. Rotational invariant Distributions	115
5.4. Convex Hulls in $\mathbb{R}^2$	116
5.5. Percolation of Loops	122
III. A Generalisation of the Pólya Urn Schemes: the Pólya Sum Process	127
6. The Pólya Sum Processes	129
6.1. The Definition of the Pólya Sum Process	130

6.2. Laplace Functionals	132
6.3. Disintegration and Partial Integration	135
7. Limit Theorems for Conditioned Pólya Sum Processes	139
7.1. The Turret Ensemble	140
7.2. The Brick Ensemble	142
7.3. The General Ensemble	145
7.4. Large Deviations	147
8. Concluding Remarks	155