

An Analytic Table of Contents
in which are listed
definitions, rules, lemmas, theorems and important terms

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