

# Contents

Introduction	1
Part I: Euclidean Geometry	3
The Linear Groups	3
The Relationship Between $O(n)$ and $GL(n, \mathbf{R})$	5
Affine Subspaces and Affine Independence	7
Isometries of $\mathbf{R}^n$	9
Isometries of $\mathbf{R}^2$	13
Isometries of $\mathbf{R}^3$	15
Some Subsets of $\mathbf{R}^3$	17
Finite Groups of Isometries	21
The Platonic Solids	23
Duality	27
The Symmetry Groups of the Platonic Solids	28
Finite Groups of Rotations of $\mathbf{R}^3$	35
Crystals	37
Rotations and Quaternions	42
Problems	45
Part II: Projective Geometry	51
Homogeneous Co-ordinates	52
The Topology of $P^1$ and $P^2$	53
Duality	57
Projective Groups	59
The Cross-Ratio	61
Fixed Points of Projectivities	62
The Elliptic Plane	62
Conics	65
Diagonalization of Quadratic Forms	67
Polarity	69
Problems	75
Part III: Hyperbolic Geometry	79
The Parallel Axiom	79
The Beltrami (or projective) Model	79
Stereographic Projection	82
The Poincaré Model	84
The Local Metric	88
Areas	89
Trigonometry	91
Hyperbolic Trigonometry	92
Lines and Polarity	94
Isometries	97
Elliptic Trigonometry	99

Problems	102
Further Reading	105
List of Symbols	106
Index	108