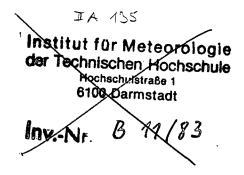
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The Method of Differential Approximation

Translated by K.G. Roesner

With 75 Figures and 12 Tables





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