TABLE OF CONTENTS

Preface. How we got from there to here. (Feferman) 1

PART A. (USES OF) THEORIES OF INDUCTIVE DEFINITIONS.

Chapter I. Iterated inductive definitions and subsystems of analysis. (Feferman, Sieg) 16

Introduction.

§1. Inductive definitions (on $\mathbb{N}$).

§1.1. General induction: operators, rule sets, and games.

§1.2. Definable operators and their iteration.

§2. Formal theories for (parts of) classical analysis.

§2.1. Theories for inductive definitions.

§2.2. Full classical analysis.

§2.3. Parts of classical analysis.

§2.4. Reduction of ID-systems to subsystems.

§3. Some proof theoretic methods and facts.

§3.1. Translations and partial truth definitions.

§3.2. Cut-elimination for number theory and ramified analysis.

§3.3. Iterated Π_1^1 -comprehension in ID-systems.

Chapter II. Proof theoretic equivalences between classical and constructive theories for analysis. (Feferman, Sieg) 78

Introduction.

§1. Reduction of subsystems of analysis to parts of T_0.

§1.1. The system T_0; elementary facts.

§1.2. Subsystems related to the ramified hierarchy.

§1.3. Subsystems related to iterated inductive definitions.

§2. Conservation results for theories with choice and comprehension principles.

§2.1. The proof theoretic basis.

§2.2. Skolem operator theories for $(\Sigma_{n+1}^1\text{-AC})\restriction$.

§2.3. Infinitary operator theories for $(\Sigma_{n+1}^1\text{-AC})$.

Appendix (concerning two formulations of operator theories).

§3. Formal models for parts of T_o in analysis.

§3.1. (Refinements of) the set theoretic model.

§3.2. Recursion theoretic models.

§3.3. Final equivalences between subsystems of analysis and of T_o.

PART B. PROOF THEORY OF THEORIES FOR INDUCTIVE DEFINITIONS WITHOUT THE USE OF SPECIAL SYSTEMS OF ORDINAL NOTATIONS.

Chapter III. Inductive definitions, Constructive ordinals, and normal derivations. (Sieg) 143

Introduction

§1. Tree classes and infinitary logic.

§1.1. The theory $ID^i_{\prec}(\mathcal{O})$ for tree classes.

§1.2. Infinitary propositional logic $PL_{\prec}$.

§2. Proof theory of $PL_{\prec}$.

§2.1. Hauptsatz and normal derivations.

§2.2. Embedding and MRP-reduction.

§3. Conservation theorems.

§3.1. Presentation of syntax.

§3.2. Logical reflection.

Chapter IV. The $\Omega_{\mu+1}$- rule. (Buchholz) 188

Introduction to the first part of Chapter IV.

§1. The formal theory ID^-_ν.

§2. The infinitary system ID^∞_ν.

§3. Majorization of deductions by abstract trees.

§4. Lower bounds for $|ID^i_\nu(\mathcal{O})|$

Introduction to the second part of Chapter IV.

§5. Reduction of ID^-_ν to a strictly positive $ID^i_\nu(\mathfrak{A})$.

§6. Realizability of strictly positive $ID^i_\nu(\mathfrak{A})$.

PART C. PROOF THEORY OF THEORIES FOR INDUCTIVE DEFINITIONS REQUIRING THE USE OF THE NOTATION SYSTEM $\overline{\Theta}(\Omega)$.

Chapter V. Ordinal analysis of ID_ν. (Buchholz) 234

Introduction.

§1. The functions $\overline{\theta}_\alpha$

§2. Majorization of abstract trees by ordinals.

§3. Constructive wellordering proofs.

Chapter VI. Proof-theoretical analysis of ID_ν by the method of local predicativity. (Pohlers) 261

Introduction.

§α. More about ordinal notations; the introduction of the collapsing functions.

§β. The infinitary system I*.

§β1. The extended language $L_{I^*}(x,y,X_i)_{i\in\mathbb{N}}$.

§β2. The notion of proof for I*.

§β3. Cut-elimination for I*.

§β4. Formalization in PA_α.

§γ. Conservative extension results.

§γ1. Some provable formulas of I*.

§γ2. Embedding of ID_ν into I*.

§γ3. Formalization in PA_α.

§γ4. Conservative extension results.

§γ5. Further results.

§δ. Characteristic ordinals for formal theories.

§δ1. Ordinal analysis for systems of iterated inductive definitions and subsystems of analysis.

§δ2. The theory AUT(ID).

§δ3. The spectrum of a formal theory.

§δ4. More about the spectrum.

Bibliography. 358

Index. 369