Contents

vii

Chapter 1

The Single First-Order Equation		;
1.	Introduction	1
2.	Examples	
3.	Analytic Solution and Approximation Methods in a Simple	-
	Example	4
	Problems	8
4.	Quasi-linear Equations	ġ
5.	The Cauchy Problem for the Quasi-linear Equation	11
6.	Examples	15
	Problems	18
7.	The General First-Order Equation for a Function of Two	
	Variables	19
8.	The Cauchy Problem	24
9.	Solutions Generated as Envelopes	29
	Problems	31
Cł	napter 2	
	econd-Order Equations: Hyperbolic Equations	
fo	or Functions of Two Independent Variables	33
1.	Characteristics for Linear and Quasi-linear Second-order	
	Equations	33
2.	Propagation of Singularities	35
3.	The Linear Second-Order Equation	37
	Problems	39
4.	The One-Dimensional Wave Equation	40
	Problems	45

viii	Contents

	 Systems of First-Order Equations A Quasi-linear System and Simple Waves Problem 	46 52 53
C	hapter 3	
C	Characteristic Manifolds and the Cauchy Problem	54
1.		54
_	Problems	55
2.	The Cauchy Problem Problems	56
3.	Real Analytic Functions and the Cauchy–Kowalevski Theorem	61
-	(a) Multiple infinite series	61 62
	Problems	63
	(b) Real analytic functions	64
	Problems (c) Analytic and real analytic functions	69
	Problems	70 72
	(d) The proof of the Cauchy-Kowalevski theorem	73
4	Problems	78
4. 5.	The University The University	79
۶.	The Uniqueness Theorem of Holmgren Problems	80
6.	Distribution Solutions	88 89
	Problems	92
Cł	napter 4	
	-	
1.	he Laplace Equation	94
1.	Green's Identity, Fundamental Solutions, and Poisson's Equation	94
	Problems	101
2.	The Maximum Principle	103
3.	Problems The Dirichlet Broklem Grand Format Format Problems	105
٥.	The Dirichlet Problem, Green's Function, and Poisson's Formula Problems	106
4.	Proof of Existence of Solutions for the Dirichlet Problem Using	110
	Subharmonic Functions ("Perron's Method")	111
_	Problems	116
5.	Solution of the Dirichlet Problem by Hilbert-Space Methods	117
	Problems	125
Ch	apter 5	
	yperbolic Equations in Higher Dimensions	126
1.	The Wave Equation in n-Dimensional Space	
• ·	(a) The method of spherical means	126 126
	Problems	132
	(b) Hadamard's method of descent	133
	Problems	134

Contents

	(c)	Duhamel's principle and the general Cauchy problem Problem	135
	(4)		139
	(d)	Initial-boundary-value problems ("Mixed" problems) Problems	139 142
2.	Hig	her-Order Hyperbolic Equations with Constant Coefficients	143
	(a)	Standard form of the initial-value problem	143
		Problem	145
	(b)	Solution by Fourier transformation	145
		Problems	156
	(c)	Solution of a mixed problem by Fourier transformation	157
	(d)	The method of plane waves	158
_	_	Problems	161
3.		ametric Hyperbolic Systems	163
	(a)	The basic energy inequality	163
	<i>a</i> \	Problems	169
	(b)	Existence of solutions by the method of finite differences	172
	(a)	Problems	181
	(c)	Existence of solutions by the method of approximation by analytic functions (Method of Schauder)	103
		tunctions (withhou of Schauder)	182
Cl	napte	r 6	
	_		
Н		er-Order Elliptic Equations with Constant	
	Co	efficients	185
1.	The	Fundamental Solution for Odd n	186
	P	roblems	188
2.	The	Dirichlet Problem	190
		roblems	195
3.	Moi	The on the Hilbert Space H_0^{μ} and the Assumption of Boundary Values in	
		Dirichlet Problem	198
	P	roblems	201
CF	apte	r 7	
	-		
Pa	arab	olic Equations	206
1.	The	Heat Equation	206
	(a)	The initial-value problem	206
	` ,	Problems	213
	(b)	Maximum principle, uniqueness, and regularity	215
		Problem	220
	(c)	A mixed problem	220
		Problems	221
	(d)	Non-negative solutions	222
	_	Problems	226
2.		Initial-Value Problem for General Second-Order Linear	
		bolic Equations	227
	(a)	The method of finite differences and the maximum principle	227
	(b)	Existence of solutions of the initial-value problem	231
		Problems	233

Contents

X

Chapter 8	
H. Lewy's Example of a Linear Equation	
without Solutions	235
Problems	239
Bibliography	241
Glossary	243
Index	245