

C O N T E N T S

CHAPTER I. MATHEMATICAL FOUNDATION OF THE STOKES PROBLEM	1
§ 1. GENERALITIES ON SOME ELLIPTIC BOUNDARY VALUE PROBLEMS.....	1
1.1. Basic concepts on Sobolev spaces	1
1.2. Abstract elliptic theory	5
1.3. Example 1 : Dirichlet's harmonic problem	6
1.4. Example 2 : Neumann's harmonic problem	8
1.5. Example 3 : Dirichlet's biharmonic problem	11
§ 2. SOME FUNCTION SPACES	12
2.1. The space $H(\text{div} ; \Omega)$	13
2.2. The space $H(\vec{\text{curl}} ; \Omega)$	19
§ 3. A DECOMPOSITION OF VECTOR FIELDS	22
3.1. Existence of the stream function of a divergence-free vector field	22
3.2. A decomposition of $[L^2(\Omega)]^n$	29
3.3. A decomposition of $[H_0^1(\Omega)]^n$	32
§ 4. ANALYSIS OF AN ABSTRACT VARIATIONAL PROBLEM	39
4.1. Statement and solution of the problem	39
4.2. A saddle-point approach	43
4.3. Numerical solution by regularization	45
4.4. Numerical solution by duality	47
§ 5. THEORY OF THE STOKES PROBLEM	50
5.1. The " velocity-pressure " formulation	50
5.2. The " stream function " formulation	55

CHAPTER II.	NUMERICAL SOLUTION OF THE STOKES PROBLEM. A CLASSICAL METHOD ..	58
§ 1.	<i>AN ABSTRACT APPROXIMATION RESULT</i>	58
§ 2.	<i>A FIRST METHOD FOR SOLVING THE STOKES PROBLEM</i>	64
2.1.	The general approximation	64
2.2.	Example 1 : a first-order approximation	70
2.3.	Example 2 : a second-order approximation	78
2.4.	Numerical solution by regularization	83
CHAPTER III.	A MIXED FINITE ELEMENT METHOD FOR SOLVING THE STOKES PROBLEM .	86
§ 1.	<i>MIXED APPROXIMATION OF AN ABSTRACT PROBLEM</i>	86
1.1.	A mixed variational problem	86
1.2.	Abstract mixed approximation	89
§ 2.	<i>APPLICATION TO THE HOMOGENEOUS STOKES PROBLEM</i>	93
2.1.	A mixed formulation of Stokes equations	93
2.2.	A mixed method for Stokes problem	98
2.3.	Application to finite elements of degree ℓ	101
CHAPTER IV.	THE STATIONARY NAVIER-STOKES EQUATIONS	104
§ 1.	<i>A CLASS OF NON-LINEAR PROBLEMS</i>	104
§ 2.	<i>APPLICATION TO THE NAVIER-STOKES EQUATIONS</i>	110
2.1.	Some results of functional analysis	110
2.2.	Solutions of the Navier-Stokes problem	113
§ 3.	<i>A FIRST METHOD FOR APPROXIMATING THE NAVIER-STOKES EQUATIONS</i>	121
3.1.	The uniqueness case	121
3.2.	The non-uniqueness case	127
§ 4.	<i>A MIXED METHOD FOR APPROXIMATING THE NAVIER-STOKES PROBLEM</i>	135
4.1.	A mixed formulation	135
4.2.	An abstract mixed approximation	138
4.3.	Applications	143

CHAPTER V. THE TIME-DEPENDENT NAVIER-STOKES EQUATIONS	148
§ 1. THE CONTINUOUS PROBLEM	148
1.1. Some vector-valued function spaces	148
1.2. Formulation of the Navier-Stokes problem	154
1.3. Existence and uniqueness of the solution	161
§ 2. NUMERICAL SOLUTION BY SEMI-DISCRETIZATION : A ONE-STEP METHOD ...	170
§ 3. SEMI-DISCRETIZATION WITH A MULTISTEP METHOD	179
3.1. Generalities about multistep methods	179
3.2. Multistep methods for solving the Navier-Stokes problem	182
3.3. Convergence of a family of two-step methods	185
BIBLIOGRAPHICAL NOTES	192
REFERENCES	195
INDEX	200