## **CONTENTS**

PART 1	NOTIONS ABOUT BOUNDARY VALUE PROBLEMS	1
Chapter 1.	DISTRIBUTIONS AND SOBOLEV SPACES	1
	1 Distributions	1
	2 Sobolev spaces	2
	3 Traces and embedding theorems	4
Chapter 2.	OPERATORS IN BANACH SPACES	7
	1 Strong, weak and weak-star topologies	7
	2 Operators in Banach spaces	9
	3 Selfadjoint operators	11
	4 Resolvent, spectrum and spectral families	13
	5 Sesquilinear forms and associated operators	17
	6 Explicit description in a particular case	21
Chapter 3.	EXAMPLES OF BOUNDARY VALUE PROBLEMS	24
	1 The Dirichlet problem for the Laplace equation	24
	2 The Neumann problem	26
	3 A transmission problem	28
Chapter 4.	SEMIGROUPS AND LAPLACE TRANSFORM	31
	1 Semigroups. Definitions and generalities	31
	2 Contraction semigroups	32
	3 Miscellaneous properties of semigroups	34
	4 Examples of "parabolic" equations	36
	5 Examples of "hyperbolic" equations	37
	6 Laplace transforms	40
PART II	HOMOGENIZATION IN THE PHYSICS OF COMPOSITE MATERIALS	45
Chapter 5.	HOMOGENIZATION OF SECOND ORDER EQUATIONS	45
	1 Formal expansion	45
	2 Study of the local problem	51
	3 Formulae for the homogenized coefficients and their	
	properties	54
	4 Proof of the convergence	57

5 Generalization to other elliptic problems and convergence	
of the resolvents	60
6 Homogenization of evolution equations	64
7 Homogenization of a boundary in heat transfer theory.	
Formal expansion	68
8 Proof of the convergence	74
9 Asymptotic expansion of an integral identity	77
10 Method of the conservation law	80
11 Comments and bibliographical notes	82
Chapter 6. HOMOGENIZATION IN ELASTICITY AND ELECTROMAGNETISM	84
1 A model problem in elastostatics	84
2 Homogenization in elasticity	88
3 A problem with couple-wise applied forces	94
4 Homogenization in viscoelasticity	97
5 Fissured elastic body. Generalities	106
6 Homogenization of an elastic body with small, periodic	
fissures	109
7 Study of the homogenized strain-stress law and consequences	112
8 Viscoelastic fissured body. Hidden variables	117
9 The Maxwell system. Asymptotic expansion	120
10 Proof of the convergence	124
11 Comments and bibliographical notes	128
	129
Chapter 7. FLUID FLOW IN POROUS MEDIA	129
1 Notions about the Stokes equations	134
2 Asymptotic expansions for flow in porous media	140
3 Effects of compressibility	140
4 Non linear effects	
5 Considerations about boundary conditions and boundary layer	149
6 Acoustics in porous media	154
7 Thermal effects	157
8 Comments and bibliographical notes	137
Chapter 8. VIBRATION OF MIXTURES OF SOLIDS AND FLUIDS	158
<ol> <li>1 Mixture of an elastic solid and a viscous fluid. Case of</li> </ol>	
large viscosity	158
<ol><li>2 Mixture of two compressible, slightly viscous fluids</li></ol>	165
3 Proof of the convergence	172

	4	Suspension of rigid particles in a slightly viscous	
	_	compressible fluid	178
	5	Connected elastic solid with canals filled with a slightly	
	_	viscous fluid	184
	6	Comments and bibliographical notes	190
PART II	Ī	MISCELLANEOUS PERTURBATION PROBLEMS	191
Chapter	<u>9</u> .	EXAMPLES OF PERTURBATIONS FOR ELLIPTIC PROBLEMS	191
	1	A class of singular perturbations	191
	2	Example - Plate of small rigidity	194
	3	Convergence of the inverses in the norm	195
	4	The case where the limit problem is not coercive	196
	5	Exemples of the preceeding section	199
	6	A problem of perturbation of the boundary condition	202
	7	Bibliographical notes	205
Chapter	<u> 10</u> .	THE TROTTER-KATO THEOREM AND RELATED TOPICS	206
	1	The Trotter-Kato theorem	206
	2	Examples: singular perturbations for nonstationary problems.	
		Application to acoustics	209
	3	Another theorem on convergence of semiproups	212
	4	A case where the configuration space depends on $\ensuremath{\epsilon}$	213
	5	Application to a problem of singular perturbation in	
		viscoelasticity	215
	6	Conclusions and comments	218
Chapter	<u>11</u> .	SPECTRAL PERTURBATION. CASE OF ISOLATED EIGENVALUES	219
	1	Resolvent, spectrum and separation	219
		Convergence in the gap and convergence of the resolvents	
		in the norm	223
	3	Spectral perturbation of operators whose resolvents converge	
		in the norm Applications to homogenization and singular	
		perturbations	225
	4	Holomorphic families of operators	227
		Implicit eigenvalue problems. Application to singular	
		perturbation in viscoelastic vibrations	233
	6	Homogenization of a boundary. Spectral properties	236
		Notes	241

Chapter	12.	PERTURBATION OF SPECTRAL FAMILIES AND APPLICATIONS TO SELFADJOINT	
		EIGENVALUE PROBLEMS	242
	1	The Rellich's theorem	242
	2	Applications to singular perturbations	245
		Remarks about hyperbolic equations and Fourier Transform.	
		Application to a problem of homogenization	248
	4	Acoustic vibrations in a domain with very corrugated boundary	254
Chapter	<u>13</u> .	STIFF PROBLEMS IN CONSTANT AND VARIABLE DOMAINS	260
	1	A model stiff problem	260
	2	Some spectral properties of stiff problems	264
	3	Heat transfer through a narrow plate with small conductivity	267
		Heat transfer through a narrow plate with high conductivity	275
		Bibliographical notes	280
Chapter	<u> 14</u> .	AVERAGING AND TWO-SCALE METHODS	281
	1	Differential equations in a Banach space and integral	
		continuity theorem	281
	2	· The averaging method	285
		The two-scale method	286
	4	- Example. Another justification of the two-scale method	288
		- Bibliohraphical notes	292
PART I	<u>v</u>	DIFFRACTION AND RELATED PROBLEMS	293
Chapter	• 15.	GENERALITIES AND POTENTIAL METHOD	293
<u></u>	1.	- Introduction	293
		- Uniqueness theorems	298
		- Representation formula and radiation condition	300
		- Potential theory	303
		- The Neumann problem. Existence and uniqueness for real ω	309
		- The transmission and Dirichlet problems. Existence and	
	••	uniqueness for real ω	312
	7.	- Analitic continuation of the solutions for complex $\omega$ .	
	•	Scattering frequencies	315
	8.	- Solutions in space-time. Interpretation of the scattering	
		frequencies	320
	9.	- Comments and bibliographical notes	323

Chapter	<u> 16</u> .	FUNCTIONAL METHODS	325
	1	Limiting absorption method	325
	2	Absolute continuity of the spectrum	334
	3	Local energy decay and limiting amplitude	337
	4	Reduction to a problem in a bounded domain	342
	5	Bibliographical notes	348
Chapter	<u> 17</u> .	SCATTERING PROBLEMS DEPENDING ON A PARAMETER	349
	1	Scattering frequencies for acoustic resonators	349
	2	Vibration of an elastic body surrounded by a gas of small	
*		density	356
	3	Diffraction problems with narrow obstacles	362
	4	Bibliographical notes	367
APPENDI)	<u>(.</u>	Incompressible fluid flow in porous media. Convergence of the	
		homogenization process.	368
		By L. Tartar	
REFERENC	CES		378
INDEX			393
NOTATION	łS		397