Contents

| Chapter 1 | |
|---|----|
| Introductory Notions | 1 |
| 1. The Fundamental Problems: Extension, Homotopy, and | |
| Classification | 3 |
| 2. Standard Notations and Conventions | 9 |
| 3. Maps of the <i>n</i> -sphere into Itself | 13 |
| 4. Compactly Generated Spaces | 17 |
| 5. NDR-pairs | 21 |
| 6. Filtered Spaces | 27 |
| 7. Fibrations | 29 |
| Chapter II | 46 |
| CW-complexes | 40 |
| 1. Construction of CW-complexes | 48 |
| 2. Homology Theory of CW-complexes | 54 |
| 3. Compression Theorems | 70 |
| 4. Cellular Maps | 76 |
| 5. Local Calculations | 79 |
| 6. Regular Cell Complexes | 81 |
| 7. Products and the Cohomology Ring | 88 |
| Chapter III | |
| Generalities on Homotopy Classes of Mappings | 96 |

98

102

xvii

1. Homotopy and the Fundamental Group

2. Spaces with Base Points

| xviii | Contents |
|-------------------------------|----------|
| 3. Groups of Homotopy Classes | 115 |

| Groups of Homotopy Classes H-spaces H'-spaces Exact Sequences of Mapping Functors Homology Properties of H-spaces and H'-spaces Hopf Algebras | 115 116 121 127 142 149 |
|--|--|
| Chapter IV Homotopy Groups | 157 |
| Relative Homotopy Groups The Homotopy Sequence The Operations of the Fundamental Group on the Homotopy | 158 161 |
| Sequence 4. The Hurewicz Map 5. The Eilenberg and Blakers Homology Groups 6. The Homotopy Addition Theorem 7. The Hurewicz Theorems 8. Homotopy Relations in Fibre Spaces 9. Fibrations in Which the Base or Fibre is a Sphere 10. Elementary Homotopy Theory of Lie Groups and Their Coset Spaces | 164 166 170 174 178 185 194 |
| Chapter V Homotopy Theory of CW-complexes | 209 |
| The Effect on the Homotopy Groups of a Cellular Extension Spaces with Prescribed Homotopy Groups Weak Homotopy Equivalence and CW-approximation Aspherical Spaces Obstruction Theory Homotopy Extension and Classification Theorems Eilenberg-Mac Lane Spaces Cohomology Operations | 211 216 219 224 228 235 244 250 |
| Chapter VI Homology with Local Coefficients | 255 |
| Bundles of Groups Homology with Local Coefficients Computations and Examples Local Coefficients in CW-complexes Obstruction Theory in Fibre Spaces The Primary Obstruction to a Lifting Characteristic Classes of Vector Bundles | 257 265 275 281 291 297 305 |

| Contents | xix |
|--|------------|
| | AIA |
| Chapter VII | |
| Homology of Fibre Spaces: Elementary Theory | 314 |
| 1. Fibrations over a Suspension | 316 |
| 2. The James Reduced Products | 326 |
| 3. Further Properties of the Wang Sequence | 336 |
| 4. Homology of the Classical Groups5. Fibrations Having a Sphere as Fibre | 341 |
| 6. The Homology Sequence of a Fibration | 349 |
| 7. The Blakers-Massey Homotopy Excision Theorem | 363 366 |
| Chapter VIII | |
| The Homology Suspension | 371 |
| 1. The Homology Suspension | 373 |
| 2. Proof of the Suspension Theorem | 379 |
| 3. Applications | 382 |
| 4. Cohomology Operations | 385 |
| 5. Stable Operations6. The mod 2 Steenrod Algebra | 390 |
| 7. The Cartan Product Formula | 394 |
| 8. Some Relations among the Steenrod Squares | 397 403 |
| 9. The Action of the Steenrod Algebra on the Cohomology of | 403 |
| Some Compact Lie Groups | 408 |
| Chapter IX | |
| Postnikov Systems | 415 |
| 1. Connective Fibrations | 417 |
| 2. The Postnikov Invariants of a Space | 421 |
| 3. Amplifying a Space by a Cohomology Class | 426 |
| 4. Reconstruction of a Space from its Postnikov System | 430 |
| 5. Some Examples | 437 |
| 6. Relative Postnikov Systems7. Postnikov Systems and Obstruction Theory | 443 |
| 7. Fostilikov Systems and Obstruction Theory | 449 |
| Chapter X | |
| On Mappings into Group-like Spaces | 456 |
| 1. The Category of a Space | 457 |
| 2 Hsnaces | 461 |

| Chapter X | |
|--|-----|
| On Mappings into Group-like Spaces | 456 |
| 1. The Category of a Space | 457 |
| 2. H ₀ -spaces | 461 |
| 3. Nilpotency of [X, G] | 462 |
| 4. The Case $X = X_1 \times \cdots \times X_k$ | 465 |
| 5. The Samelson Product | 467 |
| 6. Commutators and Homology | 470 |
| 7. The Whitehead Product | 472 |
| 8. Operations in Homotopy Groups | 478 |

| .Λ | | Contents |
|----|--|----------|
| | | |
| | | |

| Chapter XI Homotopy Operations 1. Homotopy Operations 2. The Hopf Invariant 3. The Functional Cup Product 4. The Hopf Construction 5. Geometrical Interpretation of the Hopf Invariant 6. The Hilton-Milnor Theorem 7. Proof of the Hilton-Milnor Theorem 8. The Hopf-Hilton Invariants | 488 490 494 496 502 507 511 515 533 |
|--|---|
| Chapter XII Stable Homotopy and Homology 1. Homotopy Properties of the James Imbedding 2. Suspension and Whitehead Products 3. The Suspension Category 4. Group Extensions and Homology 5. Stable Homotopy as a Homology Theory 6. Comparison with the Eilenberg-Steenrod Axioms 7. Cohomology Theories | 542 544 546 550 561 571 578 594 |
| Chapter XIII Homology of Fibre Spaces | 602 |
| The Homology of a Filtered Space Exact Couples The Exact Couples of a Filtered Space The Spectral Sequence of a Fibration Proofs of Theorems (4.7) and 4.8) The Atiyah-Hirzebruch Spectral Sequence The Leray-Serre Spectral Sequence Multiplicative Properties of the Leray-Serre Spectral Sequence Further Applications of the Leray-Serre Spectral Sequence | 604 609 613 623 632 640 645 654 |
| Appendix A Compact Lie Groups | 673 |
| Subgroups, Coset Spaces, Maximal Tori Classifying Spaces The Spinor Groups The Cayley Algebra K Automorphisms of K The Exceptional Jordan Algebra 3 The Exceptional Lie Group F₄ | 674 678 680 686 690 695 701 |

| Appendix B Additive Relations | 716 |
|--|------------|
| Direct Sums and Products Additive Relations | 717 722 |
| Bibliography | 728 |

xxi

737

Contents

Index