

TABLE OF CONTENTS

GENERAL INTRODUCTION	1
I. MATHEMATICAL PRELIMINARIES	6
1.0 Introduction	6
1.1 Linear Spaces and Topologies	6
1.2 Some Basic Mathematical Tools.	12
1.3 Basis Theory	14
1.4 Problems	20
II. RANDOM ELEMENTS IN LINEAR SPACES	21
2.0 Introduction	21
2.1 Definitions and Examples of Random Elements	21
2.2 Topological Properties of Random Elements	25
2.3 Probability Properties of Random Elements	30
2.4 Problems	42
III. LAWS OF LARGE NUMBERS, UNCORRELATION, AND CONVERGENCE OF WEIGHTED SUMS OF RANDOM VARIABLES	44
3.0 Introduction	44
3.1 Laws of Large Numbers for Random Variables	45
3.2 Extensions to Separable Hilbert Spaces	49
3.3 Comparisons of Uncorrelation Concepts	54
3.4 Convergence of Weighted Sums of Random Variables	63
3.5 Problems	70
IV. LAWS OF LARGE NUMBERS IN NORMED LINEAR SPACES.	71
4.0 Introduction	71
4.1 Laws of Large Numbers for Independent, Identically Distributed Random Elements	72
4.2 Distributional Conditions and Laws of Large Numbers.	81
4.3 Beck's Convexity and the Strong Law of Large Numbers	86

4.4	Other Geometric Conditions and the Strong	
	Law of Large Numbers	96
4.5	Other Convergence Results and Extensions to	
	Fréchet Spaces	104
4.6	Problems	106
V.	CONVERGENCE OF WEIGHTED SUMS IN NORMED LINEAR SPACES . .	108
5.0	Introduction	108
5.1	Distributional Conditions and Convergence of	
	Weighted Sums	109
5.2	Convergence for Weighted Sums of Tight Random	
	Elements	120
5.3	Geometric Conditions and Convergence of Weighted	
	Sums	137
5.4	Problems	145
VI.	RANDOMLY WEIGHTED SUMS	146
6.0	Introduction	146
6.1	Identically Distributed Results	146
6.2	Convergence in the r th Mean for Randomly Weighted	
	Sums	150
6.3	Problems	152
VII.	LAWS OF LARGE NUMBERS IN $D[0,1]$	153
7.0	Introduction	153
7.1	Preliminaries	154
7.2	Weak laws of Large Numbers in $D[0,1]$	166
7.3	Strong laws of Large Numbers in $D[0,1]$	171
7.4	A Discussion of Convex Tightness	179
7.5	Problems	184

VIII. POSSIBLE APPLICATIONS	185
8.0 Introduction	185
8.1 Applications in Stochastic Processes	186
8.2 Applications in Decision Theory	190
8.3 Applications in Quality Control	195
8.4 Applications in Estimation Problems	196
BIBLIOGRAPHY	202
SUBJECT INDEX	212