Contents

1 Introduction to Permutations, Markov Chains, and Partitions 1
1.1 Permutations and Their Matrix Representations 1
1.2 Permutation Orbits and Fixed Points 3
1.3 Fixed Points and the Inclusion-Exclusion Principle 5
1.4 Finite Markov Chains 8
1.5 Birkhoff–von Neumann Theorem 9
1.6 Generating Functions 10
1.7 Partitions 13
1.7.1 Compositions 13
1.7.2 Multi-set Permutations 14
1.7.3 Weak Partitions 15
1.7.4 Integer Partitions 16
1.8 Concluding Remarks and Further Reading 17

2 Worth Another Binary Relation: Graphs 19
2.1 Binary Relations and Their Graphs 19
2.2 Representation of Graphs by Matrices 20
2.3 Algebraic Properties of Adjacency Operators 23
2.4 Perron–Frobenius Theory for Adjacency Matrices 24
2.5 Spectral Decomposition of Adjacency Operators 26
2.6 Adjacency and Walks on a Graph 28
2.7 Principal Invariants of the Graph Adjacency Matrix 30
2.8 Euler Characteristic and Genus of a Graph 33
2.9 Euler Characteristics and Genus of Complex Networks 35
2.10 Coloring a Graph 36
2.11 Shortest Paths in a Graph 38
2.12 Concluding Remarks and Further Reading 41

Bibliografische Informationen
http://d-nb.info/1009990799

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

3 Permutations Sieved Through Adjacency: Graph Automorphisms 43
3.1 Graph Automorphisms 43
3.2 Nontrivial Graph Automorphisms and the Structure of Eigenvectors of the Adjacency Matrix 45
3.3 Automorphism Invariant Linear Functions of a Graph 47
3.3.1 Automorphism Invariant Stochastic Processes 48
3.3.2 Automorphism Invariant Harmonic Functions 49
3.4 Relations Between Eigenvalues of Automorphism Invariant Linear Functions 51
3.5 Summary 54

4 Exploring Undirected Graphs by Random Walks 55
4.1 Graphs as Discrete Time Dynamical Systems 56
4.2 Generating Functions of the Transition Probabilities 57
4.3 Cayley-Hamilton's Theorem for Random Walks 58
4.4 Stationary Distribution and Recurrence Time of Random Walks .. 59
4.5 Entropy of Random Walks Defined on a Graph 61
4.6 Hyperbolic Embeddings of Graphs by Transition Eigenvectors ... 64
4.7 Exploring the Shape of a Graph by Random Currents 68
4.8 Summary 72

5 Embedding of Graphs in Probabilistic Euclidean Space 73
5.1 Methods of Generalized Inverses in the Study of Graphs 73
5.2 Affine Probabilistic Geometry of Pseudo-inverses 75
5.3 Reduction to Euclidean Metric Geometry 76
5.4 Probabilistic Interpretation of Euclidean Geometry 77
5.5 Probabilistic Embedding of Simple Graphs 79
5.6 Group Generalized Inverse of the Laplace Operator for Directed Graphs 81
5.7 Summary 83

6 Random Walks and Electric Resistance Networks 85
6.1 Electric Resistance Network and its Probabilistic Interpretation .. 85
6.2 Dissipation and Effective Resistance in Electric Resistance Networks 87
6.3 Effective Resistance is Bounded Above by the Shortest Path Distance 89
6.4 Kirchhoff and Wiener Indexes of a Graph 90
6.5 Relation Between Effective Resistances and Commute Times 90
6.6 Summary 91

7 Random Walks and Diffusions on Directed Graphs and Interacting Networks 93
7.1 Random Walks on Directed Graphs 93
7.1.1 A Time Forward Random Walk 94
7.1.2 Backward Time Random Walks 94
7.1.3 Stationary Distributions of Random Walks on Directed Graphs 95
7.2 Laplace Operator Defined on Aperiodic Strongly Connected Directed Graphs 96
7.2.1 Bi-orthogonal Decomposition of Random Walks Defined on Strongly Connected Directed Graphs 98
7.3 Spectral Analysis of Self-adjoint Operators Defined on Directed Graphs 101
7.4 Self-adjoint Operators Defined on Interacting Networks 103
7.5 Summary 105

8 Structural Analysis of Networks and Databases 107
8.1 Structure and Function in Complex Networks and Databases 108
8.2 Graph Cut Problems 109
8.2.1 Weakly Connected Graph Components 110
8.2.2 Graph Partitioning Objectives as Trace Optimization Problems 112
8.3 Markov Chains Estimate Land Value in Cities 116
8.3.1 Spatial Networks of Urban Environments 117
8.3.2 Spectra of Cities 118
8.3.3 First-passage Times to Ghettos 120
8.3.4 Random Walks Estimate Land Value in Manhattan 121
8.4 Unraveling the Tangles of Language Evolution 123
8.4.1 Applying Phylogenetic Methods to Language Taxonomies 124
8.4.2 The Data Set We Have Used 125
8.4.3 The Relations Among Languages Encoded in the Matrix of Lexical Distances 126
8.4.4 The Structural Component Analysis on Language Data 128
8.4.5 Principal Structural Components of the Lexical Distance Data 131
8.4.6 Geometric Representation of the Indo-European Family 132
8.4.7 In Search of Lost Time 135
8.4.8 Evidence for Proto-Indo-Europeans 137
8.4.9 In Search of Polynesian Origins 140
8.4.10 Geometric Representation of Malagasy Dialects 144
8.4.11 Austronesian Languages Riding an Express Train 148

8.5 Markov Chain Analysis of Musical Dice Games 152
8.5.1 Musical Dice Game as a Markov Chain 153
8.5.2 Encoding of a Discrete Model of Music (MIDI) into a Transition Matrix 156
8.5.3 Musical Dice Game as a Generalized Communication Process 160
8.5.4 First Passage Times to Notes Resolve Tonality of Musical Dice Games 164
8.5.5 First Passage Times to Notes Feature a Composer 167
8.6 Summary 170

9 When Feedbacks Matter: Epidemics, Synchronization, and Self-regulation in Complex Networks 171
9.1 Susceptible-Infected-Susceptible Models in Epidemics 172
9.1.1 Dynamical Equation of the Epidemic Spreading in Scale Free Networks 172
9.1.2 Simplified Equation for Low Infection Rates 174
9.1.3 Stationary Solution of the Epidemic Equation for Low Infection Rates 175
9.1.4 Dynamical Solution of the Evolution Equation for Low Infection Rates 178
9.2 Epidemic Spreading in Evolutionary Scale Free Networks 180
9.3 Transitions to Intermittency and Collective Behavior in Randomly Coupled Map Networks 183
9.3.1 The Model of Random Networks of Coupled Maps 185
9.3.2 Spatiotemporal Intermittency and Collective Behavior ... 186
9.3.3 The Evolution of $\mathbb{G}(N, k)$ with k 193
9.4 Thermodynamics of Random Networks of Coupled Maps 196
9.5 Large Gene Expression Regulatory Networks 202
9.5.1 A Model of a Large Gene Expression Regulatory Networks 203
9.5.2 Numerical Analysis of Large Gene Expression Regulatory Networks 206
9.6 Mean Field Approach to the Large Transcription Regulatory Networks 213
9.7 Summary 217

10 Critical Phenomena on Large Graphs with Regular Subgraphs 219
10.1 Description of the Model and the Results 221
10.2 The Regular Subgraphs Viewed as Riemann Surfaces 222
10.3 Nonlinear Diffusions Through Complex Networks 224

10.4 Diffusion as a Generalized Brownian Motion 229
10.5 Scaling of a Scalar Field Coupled to a Complex Network 233
10.6 Summary 235

References 237

Glossary of Graph Theory 258

Index 259