

Table of Contents

First Part. Preliminary Facts

§ 1. Sets, Categories, Topology	1
1.1. Sets	1
1.2. Categories and Functors	3
1.3. The Elements of Topology	6
§ 2. Groups and Homogeneous Spaces	11
2.1. Transformation Groups and Abstract Groups	11
2.2. Homogeneous Spaces	15
2.3. Principal Types of Groups	16
2.4. Extensions of Groups	18
2.5. Cohomology of Groups	21
2.6. Topological Groups and Homogeneous Spaces	22
§ 3. Rings and Modules	24
3.1. Rings	25
3.2. Skew Fields	27
3.3. Modules over Rings	28
3.4. Linear Spaces	31
3.5. Algebras	32
§ 4. Elements of Functional Analysis	35
4.1. Linear Topological Spaces	35
4.2. Banach Algebras	44
4.3. C^* -Algebras	48
4.4. Commutative Operator Algebras	52
4.5. Continuous Sums of Hilbert Spaces and von Neumann Algebras	57
§ 5. Analysis on Manifolds	62
5.1. Manifolds	62
5.2. Vector Fields	68
5.3. Differential Forms	74
5.4. Bundles	77
§ 6. Lie Groups and Lie Algebras	83
6.1. Lie Groups	83
6.2. Lie Algebras	86
6.3. The Connection between Lie Groups and Lie Algebras	95
6.4. The Exponential Mapping	101

Second Part. Basic Concepts and Methods of the Theory of Representations

§ 7. Representations of Groups	108
7.1. Linear Representations	108
7.2. Representations of Topological Groups in Linear Topological Spaces	111
7.3. Unitary Representations	112
§ 8. Decomposition of Representations	115
8.1. Decomposition of Finite Representations	115
8.2. Irreducible Representations	119
8.3. Completely Reducible Representations	121
8.4. Decomposition of Unitary Representations	125
§ 9. Invariant Integration	129
9.1. Means and Invariant Measures	129
9.2. Applications to Compact Groups	133
9.3. Applications to Noncompact Groups	138
§ 10. Group Algebras	139
10.1. The Group Ring of a Finite Group	139
10.2. Group Algebras of Topological Groups	141
10.3. Application of Group C^* -Algebras	145
10.4. Group Algebras of Lie Groups	148
10.5. Representations of Lie Groups and their Group Algebras	152
§ 11. Characters	156
11.1. Characters of Finite-Dimensional Representations	156
11.2. Characters of Infinite-Dimensional Representations	160
11.3. Infinitesimal Characters	163
§ 12. Fourier Transforms and Duality	166
12.1. Commutative Groups	167
12.2. Compact Groups	173
12.3. Ring Groups and Duality for Finite Groups	177
12.4. Other Results	179
§ 13. Induced Representations	181
13.1. Induced Representations of Finite Groups	182
13.2. Unitary Induced Representations of Locally Compact Groups	187
13.3. Representations of Group Extensions	195
13.4. Induced Representations of Lie Groups and their Generaliza- tions	199
13.5. Intertwining Operators and Duality	205
13.6. Characters of Induced Representations	209
§ 14. Projective Representations	215
14.1. Projective Groups and Projective Representations	215
14.2. Schur's Theory	220
14.3. Projective Representations of Lie Groups	222

§ 15. The Method of Orbits	226
15.1. The Co-Adjoint Representation of a Lie Group	226
15.2. Homogeneous Symplectic Manifolds	231
15.3. Construction of an Irreducible Unitary Representation by an Orbit	235
15.4. The Method of Orbits and Quantization of Hamiltonian Mechanical Systems	241
15.5. Functorial Properties of the Correspondence between Orbits and Representations	249
15.6. The Universal Formula for Characters and Plancherel Measures	251
15.7. Infinitesimal Characters and Orbits	256

Third Part. Various Examples

§ 16. Finite Groups	259
16.1. Harmonic Analysis on the Three-Dimensional Cube	259
16.2. Representations of the Symmetric Group	262
16.3. Representations of the Group $SL(2, \mathbb{F}_q)$	265
16.4. Vector Fields on Spheres	268
§ 17. Compact Groups	271
17.1. Harmonic Analysis on the Sphere	271
17.2. Representations of the Classical Compact Lie Groups	274
17.3. Spinor Representations of the Orthogonal Group	276
§ 18. Lie Groups and Lie Algebras	279
18.1. Representations of a Simple Three-Dimensional Lie Algebra	279
18.2. The Weyl Algebra and Decomposition of Tensor Products	281
18.3. The Structure of the Enveloping Algebra $U(\mathfrak{g})$ for $\mathfrak{g} = \mathfrak{sl}(2, \mathbb{C})$	283
18.4. Spinor Representations of the Symplectic Group	287
18.5. Representations of Triangular Matrix Groups	290
§ 19. Examples of Wild Lie Groups	292
A Short Historical Sketch and a Guide to the Literature.	297
Bibliography	302
Subject Index	310