

INTRODUCTIONP A R T ISPIN $\frac{1}{2}$ LATTICE SYSTEMS WITHOUT CONSTRAINTS

1.	<u>DEFINITIONS AND GROUP STRUCTURE</u>	1
1.1.	Definition of Spin Lattice Systems	1
1.2.	Thermodynamics and Correlation Functions	2
1.3.	Group Structure for Finite Lattice Systems	3
1.4.	Group Structure for Infinite Systems	10
2.	<u>THE DUALITY TRANSFORMATION</u>	13
2.1.	The Duality Relation	13
2.2.	The High and Low Temperature Expansion	14
2.3.	Duality Transformation for Finite Systems and Duality Relation for the Partition Function	16
2.4.	General Method to Construct a Dual Lattice	20
2.5.	General Properties of the Mapping d	21
2.6.	Duality Transformation for General Systems	22
2.7.	Some Examples	26
2.8.	Concluding Remarks and Summary of Results	34
2.9.	Summary of Duality Transformation	37
3.	<u>DUALITY RELATION FOR THE CORRELATION FUNCTIONS</u>	38
3.1.	Equilibrium State and Boundary Conditions	38
3.2.	Duality Relations for Correlation Functions of Finite Systems	42
3.3.	Symmetric States and Symmetric Algebra	47
3.4.	Applications	51

4.	<u>PHASE TRANSITIONS WITH SPONTANEOUS SYMMETRY BREAKDOWN. ERGODIC DECOMPOSITION</u>	59
4.1.	Introductory Remarks	59
4.2.	Internal and Euclidean Symmetry Group for Infinite Systems. Ergodic Decomposition	60
4.3.	Generalized Peierls Argument	69
4.4.	Application	76
4.5.	Invariant States and States with Free Boundary Conditions	80
4.6.	Phase Transitions without Breakdown of the Internal Symmetry Group	87
5.	<u>SERIES AND CLUSTER EXPANSION</u> <u>AN APPLICATION OF UNIVERSALITY HYPOTHESIS</u> <u>A "GENERALIZED" DROPLET-MODEL</u>	95
5.1.	Introduction	95
5.2.	Ratio method	96
5.3.	Remark about Universality and the Critical Behaviour of the Triangular Model	101
5.4.	A Generalized Droplet-Model	105
5.5.	Analytic Properties of the Generalized Droplet- Model	116
6.	<u>THE PARTIAL TRACE TRANSFORMATION</u> <u>EQUATION FOR THE CORRELATION FUNCTIONS AND</u> <u>REPRESENTATION OF THE SYMMETRY GROUP</u>	122
6.1.	Partial Trace Transformation	122
6.2.	Equation for the Correlation Functions	128
6.3.	Representation of the Symmetry and Translation Group ($\mathcal{J}$ resp. $\mathcal{T}$)	133
6.4.	Application	137
7.	<u>INVARIANT EQUILIBRIUM STATES AND DUALITY</u> <u>TRANSFORMATION FOR INFINITE SYSTEMS</u>	152
7.1.	Symmetric Equilibrium States	152
7.2.	Symmetric Equilibrium States and Duality	156
7.3.	Duality Transformation and Surface Tension	159

8.	<u>ASANO CONTRACTION AND GROUP STRUCTURE</u>	
	<u>ANALYTICITY PROPERTIES OF THE FREE ENERGY</u>	161
8.1.	Introduction	161
8.2.	Asano Contraction for Polynomial Associated with a Group $G \subset P(B)$	163
8.3.	Properties of Small Polynomials	166
8.4.	Analyticity Properties of the Free Energy	169
8.5.	Examples	172
8.6.	Conclusions	174
9.	<u>ANALYTICITY AND UNIQUENESS OF THE INVARIANT EQUILIBRIUM STATE</u>	175
9.1.	Introduction	175
9.2.	Unicity of the $\mathbb{Z}^{\nu}$ -invariant equilibrium state at high temperatures	175
9.3.	Unicity of the invariant equilibrium state at low temperature for ferromagnetic systems	177
9.3.	Comparison with other results	179

P A R T II

SPIN $\frac{1}{2}$ LATTICE SYSTEMS WITH CONSTRAINTS

1.	<u>DEFINITIONS AND GROUP STRUCTURE FOR SYSTEMS WITH CONSTRAINTS</u>	180
1.1.	Definition and General Properties	180
1.2.	Examples of Systems with Constraints	183
1.2.1.	Systems Satisfying the Subgroup Property	184
1.2.2.	General Hard Core Lattice Systems	185
1.2.3.	k-Vertex Models, $k \leq 16$	187
1.3.	Thermodynamics, Gibbs States and some Properties of Finite and Infinite Systems	192
1.4.	Group Structure for Systems with Constraints	194
2.	<u>EXPANSIONS FOR THE PARTITION FUNCTION</u>	196
2.1.	The Low Temperature (L.T.) Expansion	196
2.2.	The High Temperature (H.T.) Expansion	198

3.	<u>PARTIAL TRACE METHOD AND EQUILIBRIUM EQUATIONS</u>	202
3.1.	Partial Trace Transformation	202
3.1.1.	General Framework	202
3.1.2.	Partial Trace Method for General Hard Core Systems	204
3.1.3.	Partial Trace Method Applied to Systems Satisfying the Subgroup Property	206
3.2.	Equations for the Correlation Functions and Equilibrium Equations	209
4.	<u>DUALITY TRANSFORMATION RESTRICTED TO FINITE BONDS</u>	212
4.1.	Duality Transformations for Systems Satisfying the Subgroup Property	212
4.2.	On the Definition of Phase Transitions	218
5.	<u>ASANO CONTRACTIONS AND UNICITY OF STATE</u>	222
5.1.	Systems Satisfying the Subgroup Property	223
5.2.	Systems Without the Subgroup Property	229
5.2.1.	Covering Sets Satisfying Asano Condition	230
5.2.2.	Systems with $\bar{\mathcal{B}} \supset \mathcal{B}_\infty$ or $\phi \in \mathcal{P}^*(\mathcal{L})$	232
5.2.3.	General Hard Core Lattice Systems	238
5.2.4.	Systems with $\bar{\mathcal{B}} \not\supset \mathcal{B}_\infty$	243
	APPENDIX	245

P A R T I I I

ARBITRARY SPIN LATTICE SYSTEMS

1.	<u>GENERAL FRAMEWORK OF HIGHER SPIN SYSTEMS</u>	250
1.1.	Definition of Arbitrary Spin Systems; Physical Picture and Group Picture	250
1.2.	Thermodynamic and Equilibrium States of Finite Systems	254

1.3.	Group Structure for Finite and Infinite q -Component Systems	257
1.4.	Group Structure for General Spin Systems	262
2.	<u>PHYSICAL IMPLICATONS OF THE GROUP STRUCTURE</u>	265
2.1.	Low Temperature Expansion	266
2.2.	High Temperature Expansion	267
2.3.	Poisson Formulae	268
2.4.	Symmetry Properties	270
2.4.1.	Internal Symmetry Group $\mathcal{S}$	270
2.4.2.	Symmetric States and Symmetric Algebra	271
2.4.3.	Permutation Group	272
2.5.	Correlation Functions	275
2.6.	Relation between Physical Picture and Group Picture	278
2.7.	Example : Generalized Potts Model	280
3.	<u>SPIN 1 LATTICE SYSTEMS</u>	283
3.1.	General Group Structure	283
3.2.	Action of the Permutation Group $\mathcal{S}_3$	286
3.3.	Symmetry Properties of the Hamiltonian	287
3.4.	Some Exact Phase Diagrams of Spin 1 Models	290
4.	<u>THE DUALITY TRANSFORMATION</u>	293
4.1.	Introduction	293
4.2.	Duality Transformation and Duality Relation for the Partition Function	294
4.3.	General Method to Construct a Dual Lattice	296
4.4.	Examples	297
5.	<u>ZEROES OF THE PARTITION FUNCTION</u>	304
5.1.	General Formalism	305
5.2.	Applications to Various Spin 1 Models	306