

Contents

Introduction	1
1. The Motion of the Human Knee as a Dynamical Contact Problem	5
1.1. The Human Knee	5
1.2. Basics of the Mathematical Model	6
1.3. Strong and Variational Problem	11
1.4. Conservation Properties and Persistency Condition	16
2. Newmark Methods	23
2.1. Classical Newmark Method	25
2.2. Contact-Implicit Newmark Method	28
2.3. Contact-Stabilized Newmark Method	29
2.4. Improved Contact-Stabilized Newmark Method	32
2.5. Numerical Comparison	36
3. A Perturbation Result – Viscosity and Physical Energy Norm	45
3.1. Elasticity	46
3.2. Viscoelasticity	48
3.3. Interpretation of the Stability Condition	53
3.4. Approximations of the Signorini Condition	57
4. Consistency – Bounded Variation	59
4.1. Newmark Methods in Function Space	59
4.2. Consistency Error in Physical Energy Norm	66
4.3. Consistency Error in a Discrete Displacement Norm	78
4.4. Consistency Error for Permanent Active Contact	82
5. Convergence Theory	91
5.1. Discrete Perturbation Results	91
5.1.1. A Discrete Perturbation Result in Discrete Displacement Norm	92
5.1.2. Discrete Stability Condition	94
5.1.3. A Discrete Perturbation Result in Physical Energy Norm . . .	99
5.2. Convergence	103
5.2.1. Convergence in Physical Energy Norm	106

5.2.2. Convergence in Discrete Displacement Norm	108
6. Adaptive Timestep Control	111
6.1. Towards an Asymptotic Error Expansion	111
6.1.1. Conical Derivative	112
6.1.2. Extension of Extrapolation Techniques	114
6.1.3. Construction of a Higher-Order Scheme	116
6.1.4. Discussion of Consistency Order	124
6.2. Timestep Control	127
6.2.1. Error Estimator in the Absence of Contact	128
6.2.2. Error Estimator in the Presence of Contact	129
6.2.3. Combined Timestep Strategy	131
6.3. Global Discretization Error	134
7. Numerical Results	137
7.1. Hertzian Contact Problem	137
7.2. Motion of the Human Knee	142
Conclusion	149
A. Inequalities	151
B. Interpretation of Contact Stresses	153
B.1. An Abstract Trace Theorem	153
B.2. Localization on Active Contact Boundaries	154
B.3. Localization on Critical Contact Boundaries	156
B.4. Interpretation of Discrete Contact Stresses	158
List of Symbols	i
Bibliography	xii
Danksagung	xiii
Zusammenfassung	xv