

Contents

Preface to the English Edition — V

Introduction — XIX

I Analytical Part: Analysis of Logical Languages

Section One

The Language of the Syllogistic — 3

- 1 The Language of the Assertoric Syllogistic — **3**
 - § 1 Syllogistic Propositional Schemata — **3**
 - § 2 In Search of a Universal Language of Deductive Logic — **5**
 - § 3 The Logical Vocabulary of the Assertoric Syllogistic — **6**
 - § 4 Additions to the Logical Vocabulary of the Assertoric Syllogistic — **8**
 - § 5 Categorical Form — **12**
- 2 The Language of the Modal Syllogistic — **15**
 - § 6 The Logical Vocabulary of the Modal Syllogistic — **15**
 - § 7 The Indispensability of the Logical Vocabulary of the Modal Syllogistic — **22**
- 3 Analysis of Expressions of Negation — **24**
 - § 8 The Bivalence Principle — **24**
 - § 9 The Principle of Excluded Middle — **25**
 - § 10 Truth-Functional and Non-Truth-Functional Negation — **27**
 - § 11 The Strong Logical Square of Assertoric Opposition — **31**
 - § 12 The Weak Logical Square of Assertoric Opposition — **33**
 - § 13 The Logical Square of Modal Opposition — **34**
- 4 Symbolic Abbreviations — **36**
 - § 14 Syllogistic and Non-Syllogistic Modal Expressions — **36**
 - § 15 Other Expressions in the Language of Elementary Syllogistic — **38**

Section Two

The Language of the Calculus of Classes — 41

- 1 A Translation Program — **41**
 - § 16 The Vocabulary of the Calculus of Classes — **41**
 - § 17 The Strong Logical Square of Assertoric Opposition in the Language of the Calculus of Classes — **43**

- 2 The Problem of Implicit Existence Assumptions — **46**
 - § 18 The Problem of the Empty Class — **46**
 - § 19 Alternative Theories — **47**
- 3 Limits of the Language of the Calculus of Classes — **50**
 - § 20 Grounds for a Modification of the Translation Program — **50**
 - § 21 The Null Class and Empty Conceptual Scopes — **54**
 - § 22 Translation of Class-Logical Propositional Schemata into the Language of the Syllogistic — **58**
 - § 23 Reduction of Class-Logical Modes of Inference to Syllogistic Modes of Inference — **63**
 - § 24 Advantages of a Transformation of the Language of the Calculus of Classes into the Language of the Calculus of Functions — **65**

Section Three

The Language of the Logical Calculus of Functions — 72

- 1 A Translation Program — **72**
 - § 25 The Representation of Concept Expressions with Functional Expressions — **72**
 - § 26 The Function Theory of Concepts — **74**
 - § 27 Grammatical and Logical Predicates — **78**
 - § 28 Remarks on the Function Theory of Concepts — **84**
- 2 Examination of the Translation Program — **90**
 - § 29 Subordination of Concepts in the Language of the Calculus of Functions — **90**
 - § 30 Truth Functions and their Reduction to Non-truth-functional Forms — **92**
 - § 31 An Extension of the Language of the Syllogistic — **102**
 - § 32 The Categorical Form in the Language of the Calculus of Functions — **105**
 - § 33 The Logical Form of Singular Propositions — **110**
 - § 34 Quantifier Rules and the Weak Logical Square — **115**
 - § 35 The Assumption of Non-Empty Domains — **118**
- 3 Limits of the Language of the Logical Calculus of Functions — **125**
 - § 36 A Formal Language of Non-Pure Thinking — **125**
 - § 37 Reference to Intuition and Symbolic Construction — **129**
 - § 38 The Problem of Synthetic Propositions A Priori in Arithmetic — **133**

- 4 The Vocabulary of the Universal Language of Deductive Logic — **135**
 - § 39 Non-Syllogistic Basic Rules in the Language of Syllogistic — **135**
 - § 40 The Disposability of an Expression for Logical Conjunction
in the Elementary Language of Syllogistic — **139**

II Synthetic Part: Construction of the Logical Calculus of Functions from the Elements of the Syllogistic

- § 41 General Preliminary Remark — **145**

Section One

Rules for Deriving Rules — 149

- 1 Principles — **149**
 - § 42 Notation — **149**
 - § 43 Definitions — **150**
 - § 44 Basic Rules — **153**
- 2 Derived Rules — **154**
 - § 45 Rules Derived From the Basic Rules — **154**

Section Two

Hypothetical and Disjunctive Syllogistic — 158

- 1 Preliminary Remark — **158**
- 2 Principles — **159**
 - § 46 Notation — **159**
 - § 47 Definitions — **160**
 - § 48 Basic Rules — **164**
- 3 Derivation of Formulas of the Hypothetical and Disjunctive
Syllogistic — **165**
 - § 49 Derived Rules — **165**
 - § 50 Conditionalization — **174**

Section Three

Categorical Syllogistic — 178

- 1 Preliminary Remark — **178**
- 2 Principles — **178**
 - § 51 Notation — **178**
 - § 52 Definitions — **178**
 - § 53 Basic Rules — **182**
- 3 Derived Rules — **184**

- § 54 Rules of the Strong and Weak Logical Squares — **184**
- § 55 Conversion Rules — **187**
- § 56 Categorical Syllogisms — **189**
- § 57 Universally Valid Propositional Schemata — **201**

Section Four

Modal Syllogistic — 203

- 1 Preliminary Remark — **203**
- 2 Principles — **204**
 - § 58 Notation — **204**
 - § 59 Definitions — **204**
 - § 60 Basic Rules — **210**
- 3 Derived Rules — **212**
 - § 61 Modal Consequence Rules — **212**
 - § 62 Modal Conversion Rules — **219**
 - § 63 Modal Syllogisms — **225**

Section Five

Contentful Syllogistic Inference — 261

- § 64 Notation — **261**
- § 65 Comments on the Notation — **263**
- § 66 Examples of Contentful Syllogistic Inference — **264**
- § 67 Abbreviation Rules — **267**
- § 68 Examples of Contentful Syllogistic Inference (Continued) — **268**
- § 69 Further Abbreviation Rules — **270**

Section Six

**Derivation of Formulas within the Framework of a System Extended
Beyond the Limits of Elementary Deductive Logic — 271**

- 1 Principles and Metalogical Rules — **271**
 - § 70 Notation — **271**
 - § 71 Definitions — **271**
 - § 72 Postulates — **272**
 - § 73 Metalogical Rules — **275**
- 2 Derived Formulas — **277**
 - § 74 Derivation of Truth-Functional Rules — **277**
 - § 75 Derivation of Truth-Functional Laws — **281**
 - § 76 Contentful Inference — **286**

Section Seven

Provability and Derivability within the Logical Calculus of Functions — 293

§ 77 Notation — **293**

§ 78 An Axiomatic System of the Logical Calculus of Functions — **294**

§ 79 Definitions — **296**

§ 80 Theorems of the Calculus of Functions — **297**

§ 81 Soundness and Completeness — **301**

Conclusion — 306

§ 82 Principles and Rules on Which the Complete System of the
Calculus of Functions Depends — **306**

§ 83 Logical Form: A Review of the Results Attained — **308**

§ 84 Stages in the History of Logic — **310**

Appendix 1

On the Completeness of a Syllogistic without Logical Conjunction — **315**

Appendix 2

Mathematical Induction without Higher-Level Predicate Logic — **321**

Appendix 3

Reduction of Truth-Functional Expressions to Non-Truth-Functional
Expressions — **329**

Appendix 4

Compatibility and Incompatibility — **333**

Appendix 5

Modern Non-Syllogistic Systems of Modal Logic in Their Relationship to Modal
Syllogistic — **337**

Appendix 6

Non-Classical Systems of Logic in Their Relationship to the Logical Calculus of
Functions — **351**

Appendix 7

The Barcan Formula — **357**

Appendix 8

On Bivalence — **359**

Appendix 9

Absolute Logical Constants — **365**

Index of Symbols Used — 369

Outline of the Rules Used Mainly in the Proofs of Part II — 371

Index of Metalogical Rules Used in the Proofs of Part II — 373

Index of Syllogistic Rules Used Directly in the Validity Proofs of Assertoric Syllogisms (in § 56) — 375

Index of Modal-Syllogistic Rules Used Directly in the Validity Proofs of Modal Syllogisms (in § 63) — 377

Index of Logical Rules Used Directly for the Derivation of Truth-Functional Rules and Laws (in § 74 and § 75) — 379

Bibliography — 381

Subject Index — 387

Name Index — 395