

CONTENTS

<u>CHAPTER I. LOCAL GEOMETRY OR LENGTH</u> -----		1
§ 1.	General terminology-----	1
§ 2.	Principal ideals and prime ideals.-----	1
§ 3.	Total quotient ring and conductor.-----	2
	(3.1). Localization of the conductor.	
§ 4.	Normal model.-----	3
	(4.1). Divisor of a function.	
	(4.2). Divisor of zeros of a function.	
	(4.3). The " $\sum e_i f_i = n$ " formula.	
§ 5.	Length in a one-dimensional noetherian domain, or affine intersection multiplicity on an irreducible curve.-----	6
	(5.1). Values of local intersection multiplicity. Various cases.	
	(5.2). Local expansion of intersection multiplicity over a divisor.	
	(5.3). Global intersection multiplicity.	
	(5.4). Local expansion of length.	
	(5.5). Intersection multiplicity equals length in the integral closure (R^*).	
	(5.6). Local intersection multiplicity equals a length (in R) for a principal ideal.	
	(5.7). Globalization of (5.6).	
	(5.8). Special case of (5.6) - the normal case.	
	(5.9). Special case of (5.7) - the normal case.	
	(5.10). Definition. Multiplicity of a local domain of dimension one.	
	(5.11). Definition and properties. Conductor, its length; and adjoints.	
	(5.12). Lemma on overadjoints.	
§ 6.	Length in a one-dimensional noetherian homomorphic----- 20 image, or affine intersection multiplicity on an embedded irreducible curve.	
	(6.1). Values of local intersection multiplicity. Various cases.	
	(6.2). Local expansion of intersection multiplicity in the preimage.	
	(6.3). Global intersection multiplicity.	
	(6.4). Case of algebraically closed ground field	
	(6.5). Case when a curve is thought to be embedded in itself.	
	(6.6) to (6.9). Restatements of (5.6) to (5.9) for the case of a homomorphic image.	
§ 7.	A commuting lemma for length.----- 27	
	(7.1). For two embedded irreducible curves, at a common simple point, the intersection multi- plicity of either one with the other is the same.	
	(7.2). Globalization of (7.1) over a divisor.	

(7.3) Complete globalization of (7.1).

§ 8.	Length in a two-dimensional regular local domain.-----	28
(8.1).	Intersection multiplicity of curves embedded in a regular surface. Local case.	
(8.2).	For two curves embedded in a regular surface, the intersection multiplicity of either one with the other is the same. Local case.	
(8.3).	Additivity of intersection multiplicity of curves embedded in a regular surface. Local case.	
§ 9.	Multiplicity in a regular local domain.-----	30
(9.1).	Multiplicity of an irreducible curve (embedded in a regular surface) at a point is the order of its defining equation.	
(9.2).	Technical lemma for (9.1).	
§ 10.	Double points of algebraic curves.-----	33
(10.1).	Theorem. Description of a double point of a curve.	
(10.2).	Lemma. Description of high nodes.	
(10.3).	Lemma. Description of high cusps.	
(10.4).	Lemma. Description of nonrational high cusps.	
<u>CHAPTER II. PROJECTIVE GEOMETRY OR HOMOGENEOUS DOMAINS</u> -----		66
§ 11.	Function fields and projective models.-----	66
§ 12.	Homogeneous homomorphism.-----	68
§ 13.	Homogeneous ideals (projective varieties) and hypersurfaces.-----	68
§ 14.	Homogeneous subdomains, flats (linear varieties), projections, birational projections, and cones.-----	70
(14.1).	Dimension and embedding dimension of a homogeneous subdomain.	
(14.2) and (14.3).	Dimension and embedding dimension of a (homogeneous) homomorphic image.	
§ 15.	Zeroset and homogeneous localization.-----	75
(15.1), (15.2) and (15.3).	Extension to (homogeneous) localization.	
(15.4) and (15.5)	Alternative (affinized) description of the (homogeneous) localization.	
(15.6).	Correspondence between homogeneous prime ideals and homogeneous localization.	
(15.7), (15.8) and (15.9).	Restatement of (15.1), (15.2) and (15.3) for embedded varieties.	
(15.10).	Lemma. Number of conditions imposed on a linear system of hypersurfaces.	
§ 16.	Homogeneous coordinate systems.-----	85

- § 17. Polynomial rings as homogeneous domains.----- 86
 (17.1) to (17.5).
 Equivalent descriptions and properties of homogeneous domains which are polynomial rings over a field.
- § 18. Order on an embedded (irreducible) curve and integral projections.----- 88
 (18.1) and (18.2).
 Order of a hypersurface at a valuation of an embedded curve.
 (18.3) and (18.4).
 Order of an ideal at a valuation of an embedded curve.
 (18.5). Zerosets of ideals.
 (18.6) to (18.10).
 Order at a valuation of an embedded curve behaves like a valuation.
 (18.11). Projection lemma. Projection of valuation and order, from a vector space.
 (18.12). Projection lemma. Projection of valuation and order, from a flat (linear variety).
 (18.13). Corollary-definition. Condition for a π -integral projection (where π is a hyperplane).
 (18.13.1). Special case of (18.13)-projection from a center not meeting the curve.
- § 19. Order on an abstract (irreducible) curve and integral projections.----- 93
 (19.1) to (19.12).
 Versions of (18.1) to (18.12) when a curve is thought of as embedded in itself.
 (19.13) and (19.13.1).
 Versions of (18.13) and (18.13.1) for an abstract curve.
 (19.14). Remark. "Integral"ness of projection commutes with homomorphic image.
- § 20. Valued vector spaces.----- 95
 (20.1) to (20.13).
 Structure and properties of a valued vector space.
- § 21. Osculating flats and integral projections of an embedded (irreducible) curve.----- 109
 (21.1) Definition and structure of osculating flats.
 (21.2), (21.3) and (21.4).
 Application of § 20 to the properties of osculating flats.
 (21.5). Properties of osculating flats in special cases.
 (21.6). Condition for integral projection in terms of osculating flats at the center of projections.
- § 22. Osculating flats and integral projections of an abstract (irreducible) curve.----- 118
 (22.1) to (22.6).
 Restatements of (21.1) to (21.6) when a curve is thought of as embedded in itself
 (22.7). Remark. Osculating flats commute with homomorphic image.

- § 23. Intersection multiplicity with an embedded ————— 121
 (irreducible projective) curve.
 (23.1), (23.2) and (23.3).
 Properties of intersection multiplicity with
 an embedded curve.
 (23.4). Case of algebraically closed ground field.
 (23.5). Additivity of intersection multiplicity.
 (23.6). Intersection multiplicity equals length
 for a principal ideal.
 (23.7). All points of an embedded line are simple.
 (23.8) and (23.9). Bezout's Little Theorem.
 Definition. Degree of an embedded
 (irreducible) curve.
 (23.10). Remark. Affine interpretation of degree.
 (23.11). Lemma. If there are enough rational points
 then embedding dimension of an embedded curve
 is less than or equal to its degree.
 (23.12). Lemma. If the degree of an embedded curve
 is one, then its embedding dimension is one.
 (23.13). Remark. Hyperplanes have degree one.
 (23.14). Projection formula. Projection of varieties
 from flats.
 (23.15). Special projection formula. Degree of
 the projection.
 (23.16). Remark. Case of an algebraically closed
 ground field.
 (23.17). Lemma. Birationality of the projection from
 the generic point on a line.
 (23.18). Definition and properties of tangents to
 an embedded curve.
 (23.19) and (23.20). Commuting lemmas. Versions of
 (7.1) and (8.2) for projective curves.
- § 24. Intersection multiplicity with an abstract
 (irreducible) curve. ————— 138
 (24.1) to (24.17). Versions of (23.1) to (23.17)
 for an abstract curve.
 (24.18). Definition and properties of tangents
 to an abstract curve.
 (24.19). Remark. Relations between an embedded
 curve $[A, C]$ and an abstract curve A/C .
 (24.20). Lemma on overadjoints. Projective version
 of (5.12).
 (24.21). Lemma on underadjoints. Existence of
 certain type of projective underadjoints
 which are true adjoints in an affine piece.
- § 25. Tangent cones and quasihyperplanes. ————— 148
 (25.1). Definition. Leading form of a hypersurface.
 (25.2) and (25.3). Lemma-definition. Definition
 and properties of tangent-cones.
 (25.4). Definition. π -quasihyperplane.
 (25.5). Lemma. Characterization of π -quasihyperplanes.
 (25.6). Lemma. A hyperplane (different from π)
 is a π -quasihyperplane.
 (25.7). Lemma. Quadric π -quasiplanes.
 (25.8). Lemma. Version of (9.1) for projective curves.
 (25.9). Lemma. Degree of an embedded plane curve is
 the degree of its defining equation.

- (25.10). Lemma. Characterization of tangent lines of plane projective curves.
- (25.11). Definition Intersection multiplicity of two hypersurfaces.
- (25.12). Additivity of the intersection multiplicity of two hypersurfaces.
- (25.13). Lemma. Version of (8.2) for projective curves.
- (25.14). Bezout's Theorem. Intersection of two plane projective curves.

- §26. 2-equimultiple plane projections of projective space quintics. ----- 155
(For notation see beginning of §26.)
- (26.1). Lemma. Most lines through a d -fold point of an irreducible curve are d -secants. Also, if $d < \text{degree of the curve}$, then there are $(d+1)$ -chords through the point in every plane through the point.
- (26.2). Lemma. An irreducible curve of degree ≥ 2 has 2-chords in every plane.
- (26.3). Lemma. Sufficient condition for existence of 4-chords.
- (26.4). Lemma. Sufficient condition for existence of 2-secants.
- (26.5). Lemma. Projection from points on an $(n-1)$ -secant.
- (26.6). Lemma. Projection from points on 2-secants of an irreducible quartic.
- (26.7). Lemma. Projection from points on certain 3-secants.
- (26.8). Cone Lemma.
- (26.9). Plane Lemma.
- (26.10). Quadric Lemma.
- (26.11). Proposition. Detailed description of projections of curves of degree at most 5.
- (26.12). Theorem. Condensed version of (26.11) for reference.

CHAPTER III. BIRATIONAL GEOMETRY OR GENUS ----- 180

- §27. Different. ----- 180
 - (27.1). Definitions.
 - (27.2). Dedekind's formula. " $\xi_y = \sum dx$ ".
 - (27.3). Lemma. Condition for an unramified extension.
 - (27.4). Technical lemma.
 - (27.5). Lemma. Another characterization of an unramified extension.
 - (27.6). Description of an unramified extension.
 - (27.7). Lemma. Finiteness of the set of ramified primes.
 - (27.8). Definition. Separably generated function fields.
 - (27.9). Lemma. Conditions for separably generated function fields.
- §28. Differentials. ----- 189
 - (28.1). Reformulation of (27.2) in terms of divisors. Integral case.

(28.2).	Conversion formula for different. Integral case.	
(28.3) and (28.4).	Exchange lemmas.	
(28.5).	General case of (28.2).	
(28.6).	General case of (28.1).	
(28.7).	Definition. $\text{ord}_v(\alpha, x)$; (α, x) intended to be replaced by αdx .	
(28.8) to (28.10).	Lemma. (α, x) behaves αdx , so far as ord_v is concerned.	
(28.11).	Technical definition of genus.	
(28.12).	Theorem. A rational curve has genus zero	
(28.13).	Definition. Usual differentials and their properties.	
(28.14).	Usual definition of genus.	
(28.15).	Genus formulas.	
(28.16).	Remark. Alternative proof to a genus formula.	
(28.17).	Remark.	
(28.18).	Definition. Uniformizing parameter and coordinate.	
(28.19).	Lemma. Properties of uniformizing coordinates.	
(28.20).	Example. Valuations with unseparable residue fields.	
(28.21).	Remark.	
(28.22).	Lemma. (28.19) reformulated using differentials.	
§ 29.	Genus of an abstract curve.-----	220
(29.1) and (29.2).	Genus formulas for plane projective curves. Rationality of a curve of genus zero.	
(29.3).	Remark.	
(29.4).	Direct proof of rationality of a conic.	
(29.5).	Direct proof of rationality of a line.	
(29.6).	Direct computation of the genus of a cubic.	
(29.7).	Theorem. Application of (24.21) to plane projective curves of genus ≤ 1 and degree 4 or 5.	
§ 30.	Genus of an embedded curve.-----	231
(30.1) and (30.2).	Genus formulas. Rationality of a curve of genus zero.	
(30.3).	Theorem. Combined version of (29.7) and (26.12) for reference.	
<u>CHAPTER IV.</u>	<u>AFFINE GEOMETRY OR FILTERED DOMAINS</u> -----	234
§ 31.	Filtered domains. Various definitions.-----	234
§ 32.	Homogenization or taking projective completion.-----	236
(32.1).	Definition. Degree	
(32.3) to (32.8).	Properties of homogenization.	
§ 33.	Dehomogenization or taking an affine piece-----	238
(33.1).	Definition. Dehomogenization	
(33.2) to (33.4).	Properties of dehomogenization	

§34.	Relation between homogenization and dehomogenization.	241
§35.	Projection of a filtered domain.	244
	(35.1). Definition. Projection.	
	(35.2). Relation with homogenization and dehomogenization.	
	(35.3). Lemma. Conditions for integral projections.	
	(35.4). Definition. Degree, genus.	
	(35.5). Theorem. Affine version of (30.3) and (26.12).	
§36.	Complete intersections.	248
	(36.1). Definition. Complete intersections, essentially hyperplanar.	
	(36.2). Lemma. Essentially planar space curve is a complete intersection.	
	(36.3). Corollary to (36.2).	
	(36.4). Lemma. A case of complete intersection.	
	(36.5). Theorem. A sufficient condition for complete intersection.	
	(36.6). Elementary transformations.	
	(36.7). Theorem. Another sufficient condition for complete intersection.	
	(36.8). Corollary to (36.7).	
	(36.9). Main theorem of complete intersection.	
<u>CHAPTER V. APPENDIX.</u>		257
§37.	Double points of algebroid curves. An alternative treatment of most of §10.	257
§38.	Bezout's theorem. The general case.	268
§39.	Chains of euclidean curves. A generalized version of (36.7).	274
§40.	Treatments of differentials in dimension one. A short survey.	280
§41.	A generalization of Dedekind's formula about conductor and different.	281
§42.	The general adjoint condition.	285
§43.	Geometric language. Geometric motivations behind the various notations.	287
§44.	Index to notations.	295
§45.	Index to topics.	298

Interdependence of sections.

In the following, $\S b \leftarrow \S a_1, \dots, \S a_r$ means $\S a_1, \dots, \S a_r$ are directly referred to in $\S b$. Except for such references, the only other prerequisites for $\S b$ are the notations and definitions from previous sections and they may be located from the index.

§1, §2, §3, §4, §5, basic.