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Generalization of result to $|\underset{\sim}{ID}_\nu| = |\underset{\sim}{T}_\nu|$ for certain $\nu > 2$; equivalence of various $\underset{\sim}{ID}_\nu^c(A)$ with subsystems of classical analysis; unknown whether $|\underset{\sim}{ID}_\nu^c| = |\underset{\sim}{T}_\nu|$ for $\nu > 1$ (6.9.1) - Positive result: $|\underset{\sim}{ID}_{<\omega}^c| = |\underset{\sim}{ID}_{<\omega}|$ (6.9.2) - Theory $\underset{\sim}{W}$ - $\underset{\sim}{ID}_\omega^c(A)$ equivalent to classical analysis + Π_1^1 - CA (6.9.3)

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